ized, with variable t, to find P we need

$$\tfrac{1}{2}\frac{d}{dt}(x^2 + y^2) \quad = \quad x\frac{dx}{dt} + y\frac{dy}{dt} \quad = \quad 0 \qquad \text{(vi)}$$

where, on differentiating (i),

$$(ax + hy)\frac{dx}{dt} + (hx + by)\frac{dy}{dt} \quad = \quad 0. \qquad \text{(vii)}$$

Now (vi) and (vii) have a non-trivial solution in dx/dt, dy/dt if

$$\det\begin{pmatrix} ax + hy & hx + by \\ x & y \end{pmatrix} \quad = \quad 0,$$

which is a condition that

$$\begin{aligned} 1(ax + hy) - \lambda x &= 0 \\ 1(hx + by) - \lambda y &= 0 \end{aligned} \qquad \text{(viii)}$$

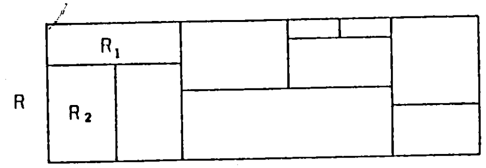
have a solution in λ. In (vi) and (vii) we are looking for a solution (x,y) which is not $(0,0)$. Then (viii) brings in the eigenvectors of (v), so with some loss of immediacy we can omit Lagrange multipliers and still reach our objective.

Department of Mathematics,
University College,
Cork.

PROBLEM PAGE

The first problem this time is 'going around' at the moment. I heard it from several different people within a period of a week, and it has a remarkable solution.

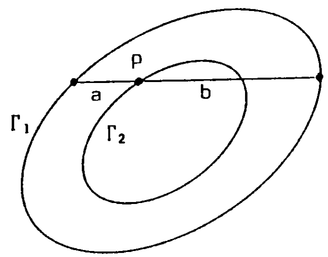
1. A rectangle R is partitioned into a finite number of rectangles $R_1, R_2, \ldots, R_n$, each of which has the property that at least one side is of integer length.



Prove that R has the same property.

The next problem came from Jim Clunie who learnt it from Tom Willmore.

2. A rod moves so that its endpoints lie on a convex curve Γ_1 in $\mathbb{R}^2$ and a point P, which divides the rod into lengths a and b, then describes a closed curve Γ_2.



Prove that the region lying between Γ_1 and Γ_2 has area πab.

Now for the solutions to two earlier problems.

1. Let A_1, A_2, A_3, A_4 be 3x3 complex matrices and let

$$M = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ A_4 & A_1 & A_2 & A_3 \\ A_3 & A_4 & A_1 & A_2 \\ A_2 & A_3 & A_4 & A_1 \end{bmatrix}.$$

Express det M as the product of four 3x3 determinants.

This problem was sent in by Finbarr Holland and also solved elegantly by Allan Solomon as follows.

Let

$$M(\lambda) = A_1 + \lambda A_2 + \lambda^2 A_3 + \lambda^3 A_4, \qquad \lambda \in C.$$

Then

$$\det M = \prod_{\lambda \in \Phi} \det M(\lambda), \text{ where } \Phi = \{1, i, -1, -i\}.$$

To see this, note that

$$M = I \otimes A_1 + T \otimes A_2 + T^2 \otimes A_3 + T^3 \otimes A_4,$$

where

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

and ' $\otimes$ ' denotes the tensor product.

The matrix T generates $\mathbb{Z}_4$ and has eigenvalues 1, i, -1, -i. Thus there is a 4x4 matrix R_0 such that

$$R_0 T R_0^{-1} = \text{diag}\{1, i, -1, -i\} = D,$$

say, and so if $R = R_0 \otimes I$ then

$$RMR^{-1} = I \otimes A_1 + D \otimes A_2 + D^2 \otimes A_3 + D^3 \otimes A_4 = \begin{bmatrix} M(1) & 0 & 0 & 0 \\ 0 & M(i) & 0 & 0 \\ 0 & 0 & M(-1) & 0 \\ 0 & 0 & 0 & M(-i) \end{bmatrix}.$$

Hence

$$\det M = \det RMR^{-1} = \prod_{\lambda \in \Phi} \det M(\lambda),$$

as required.

Allan points out that this generalises to

$$M = \sum_{n=1}^{N} T^{n-1} \otimes A_n,$$

where T generates $\mathbb{Z}_N$. Also, if $\{A_i\}$ generates a Lie algebra $\mathcal{G}$, then $\{T^k \otimes A : k \in \mathbb{Z}, A \in \mathcal{G}\}$ generates a graded Lie algebra. In an article in 'Group Theoretical Methods in Theoretical Physics' (academic Press 1977), he employed this algebra (with $\mathcal{G} = SU(2)$) to give a new solution to the Ising model on a cyclic lattice of N points.

2. Prove or give a counterexample to the following statement:

$$\text{if } a_n \geq 0, \text{ for } n = 1, 2, \ldots, \text{ and } \sum_{n=1}^{\infty} a_n < \infty \text{ then}$$

$$\sum_{n=3}^{\infty} a_n^{\left(1 - \frac{1}{\log n}\right)} < \infty.$$

In fact this statement is true. Indeed, for any term a_n such that $n \geq e^4$ and

$$a_n \leq \frac{1}{n^2},$$

we have

$$a_n^{\left(1 - \frac{1}{\log n}\right)} \leq \left(\frac{1}{n^2}\right)^{\left(1 - \frac{1}{\log n}\right)} \leq \frac{1}{n^{3/2}}.$$

On the other hand, if

$$a_n \geq \frac{1}{n^2},$$

then

$$a_n^{-\frac{1}{\log n}} \leq (n^2)^{\frac{1}{\log n}} = e^2,$$

and so

$$a_n^{1 - \frac{1}{\log n}} \leq e^2 a_n.$$

Since Σa_n and $\Sigma \frac{1}{n^{3/2}}$ are both convergent, the desired result follows.

Phil Rippon

BOOKS RECEIVED

"NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS. COLLÈGE DE FRANCE SEMINAR VOLUME VII"

Edited by *H. Brezis* and *J.L. Lions*

Published by *Pitman Publishing*, London, 1985, 292 pp.
Stg £16.50. ISBN 0-273-08679-0

This book contains the texts of selected lectures delivered at a weekly seminar held at the Collège de France. It includes contributions by leading experts from various centres on recent results in nonlinear functional analysis and partial differential equations. The emphasis is laid on applications to numerous fields including control theory, theoretical physics, fluid mechanics, free boundary value problems, dynamical systems, numerical analysis and engineering. The book will be of particular interest to postgraduate students and specialists in these areas.

"ENNIO DE GIORGI COLLOQUIUM"

Edited by *Paul Krée*

Published by *Pitman Publishing*, London, 1985, 169 pp.
Stg £14.50. ISBN 0-273-08680-4

This research note includes sixteen papers reporting mathematical research in France and Italy related to the work of Ennio de Giorgi.

In July 1983, Professor Ennio de Giorgi was awarded the title 'Doctor Honoris Causa' by the Council of the Université de Paris VI. The very profound and influential nature of his