PERSONAL ITEMS

Dr Ray Ryan of the Mathematics Department, UCG, is visiting the Department of Mathematics at Kent State University for the academic year 1985-'86.

Professor Robin Harte of the Mathematics Department, UCC, will be at the University of Iowa on sabbatical leave during the academic year 1985-'86.

Dr Gabrielle Kelly of the Statistics Department, UCC, will be at Columbia University on sabbatical leave during the academic year 1985-'86.

Dr Niall Ó Murchadha of the Department of Experimental Physics, UCC, will spend the period September 1985-March 1986 on leave at the University of British Columbia.

Mr Micheál Ó Searcóid has been appointed to a temporary lectureship at the Mathematics Department, UCC, for the academic year 1985-'86. His research interests are in Operator Theory

Dr Alastair Wood has been appointed to the Westinghouse Chair of Applied Mathematical Sciences at NIHE, Dublin.

A MATRIX JOKE

Robin Harte

1. If $x = (x_{ij}) \in A^{n,n}$ is an nxn matrix with entries x_{ij} in a ring A with identity 1, under what conditions does it have a two-sided inverse $x^{-1} \in A^{n,n}$? If the ring A is commutative, then the answer is very nearly the same as for the real or the complex numbers:

$$x \textit{ invertible in } A^{n,n} \iff |x| \textit{ invertible in } A, \qquad (1.1)$$

where $|x|$ denotes the *determinant* of x, defined [5, Chapter 5] in any one of the usual ways. If the ring A is not commutative then the formulae for the determinant become ambiguous, unless we restrict to matrices $x = (x_{ij})$ which are *commutative*, in the sense that their entries form a commutative set $\{x_{ij}\}$. With this restriction implication (1.1) was demonstrated for 2x2 matrices of Hilbert space operators by Halmos [1, Problem 55], extended to nxn matrices of Banach algebra elements using the spectral mapping theorem [3, Example 2.4], and is now given in full generality by Halmos again [2, Problem 70]. In this note we will demonstrate that (1.1) holds separately for left and right inverses, at least for 2x2 matrices: the argument seems to depend on a joke.

2. Suppose that $x = (x_{ij})$ is a commutative nxn matrix over the ring A, with determinant $|x| \in A$, and *cofactor* $x^{\sim} \in A^{n,n}$, in the sense of the usual 'adjugate' or 'classical adjoint' matrix of x: then we recall *Cramer's rule*,

$$x^{\sim}x = x x^{\sim} = |x|\underline{1}, \qquad (2.1)$$

and

$$\underline{1}^{\sim} = \underline{1},$$

where $\underline{1} = (\delta_{ij})$ is the identity matrix. If also $y = (y_{ij})$ is another commutative matrix, and if in addition *the entries of*