PROBLEM PAGE

First of all, here's a proof of the inequality

$$(1 + x^2)^p \leq x^{2p} + (2^p - 2)x^p + 1, \qquad x \geq 0,\ 1 \leq p \leq 2, \qquad (1)$$

which was posed last time.

For $x > 0$ we write

$$(x^{2p} + (2^p - 2)x^p + 1) - (1 + x^2)^p = x^p(x^p + (2^p - 2) + \frac{1}{x^p} - (x + \frac{1}{x})^p) = x^p\phi(x),$$

say. By symmetry, it is enough to prove that $\phi(x) \geq 0$ for $0 < x \leq 1$, and since $\phi(1) = 0$, we need only show that ϕ is decreasing in $(0,1)$. But

$$\phi'(x) = px^{p-1} - \frac{p}{x^{p+1}} - p(x + \frac{1}{x})^{p-1}(1 - \frac{1}{x^2})$$

so that

$$x^{p+1}\phi'(x) = p(x^{2p} - 1 + (1 - x^2)(1 + x^2)^{p-1}).$$

For $0 < x \leq 1$ we have

$$x^{2p} + (1 - x^2)(1 + x^2)^{p-1} \leq 1, \qquad 1 \leq p \leq 2.$$

since the left-hand side is convex as a function of p, and there is equality at $p = 1,2$. Thus $\phi'(x) \leq 0$ for $0 < x \leq 1$, and the proof is complete.

The idea of keeping x fixed and varying p has been used by Harold Shapiro to give a proof of (1) based on Descarte's rule of signs! Another proof of (1) depends on the expansion

$$\alpha^p = (1 - (1 - \alpha))^p = 1 - p(1 - \alpha) + \frac{p(p - 1)}{2!}(1 - \alpha)^2 - \ldots,$$

where $0 < \alpha < 1$.

The more general inequality, of which (1) is a very special case, arises in the following way. For any polynomial

$$P(z) = a_0 + a_1 z + \ldots + a_n z^n,$$

let us write

$$|P|_p = |a_0|^p + |a_1|^p + \ldots + |a_n|^p.$$

If P,Q are both polynomials of degree n with non-negative coefficients is it then true that

$$|PQ|_p^2 \leq |P^2|_p\ |Q^2|_p, \qquad 1 \leq p \leq 2? \qquad (2)$$

For $p = 1$ both sides are equal to $(P(1)Q(1))^2$ and, for $p = 2$, (2) is a form of the Schwarz inequality. All computer calculations (now done by several people) point to the truth of (2) but there are only a few positive results.

For example, if $P(z) = 1 + az$, $Q(z) = a + z$, where $a \geq 0$, then (2) reduces to (1). From (1) we can also deduce the general linear case, $P(z) = 1 + az$, $Q(z) = 1 + bz$, where $a,b \geq 0$. However the next case to consider

$$P(z) = 1 + az + bz^2,\ Q(z) = b + az + z^2,$$

where $a,b \geq 0$, has only been verified (using Shapiro's method) in certain special cases.

I hope to survey known results and the computer evidence in a future article in the *Newsletter*. For now, here are two more problems.

1. (Suggested by Finbarr Holland) Prove that

$$\sum_{k=1}^{m} \cot^2 \frac{k\pi}{2m+1} = \frac{m(2m-1)}{3},$$

and deduce that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

2. (Posed by the topologist Morton Brown) Suppose that a_1, a_2 are real and not both zero, and

$$a_{n+2} = |a_{n+1}| - a_n, \quad n = 1,2,\ldots .$$

Prove that the sequence $\{a_n\}$ always has period 9.

Phil Rippon,
Faculty of Mathematics,
The Open University,
Milton Keynes.

CONFERENCE REPORTS

FOURTH CONFERENCE ON APPLIED STATISTICS IN IRELAND

The Fourth Conference on Applied Statistics in Ireland was held in the Kilkea Castle Hotel, Castledermot, Co. Kildare, on 29-30 March 1984. This conference was the fourth in a series which brings together individuals of diverse statistical interests from industry, government and education. Fifty-three participants (including two from overseas) attended the conference and helped create an atmosphere conducive to the exchange of statistical ideas. An added bonus to this year's conference was the book displays provided by both Chapman and Hall Ltd and John Wiley and Sons Ltd. C.O.P.S. Ltd displayed IBM personal computers and some relevant statistical software.

The conference programme was divided into five sessions of contributed papers as well as two principal invited addresses. The first invited address was given by Mr Thomas P. Lenihan, Director of the Central Statistics Office. Mr Lenihan gave an overview of the C.S.O. and its activities, and one could not but be impressed by the diversity and scope of this important information collecting agency. Mr Charles Smith, chief statistician at Guinness Ireland Ltd, gave the second principal address in which he described the role of statistics at Guinness. It was quite interesting to note how diversely talented a large company like Guinness expects its statisticians to be. Although the role of the statistician in industry seems to be well appreciated (for historical and other reasons) at Guinness, it was perhaps a bit discouraging to learn that the number of statisticians employed at Guinness has decreased markedly in recent years.

The first session of contributed papers was led off by Adrian Dunne (UCD) who demonstrated the potential of an objective design strategy for pharmacokinetic model discrimination. Graham Horgan (TCD) then described some of the practical problems in the statistics of image processing, particularly with