(who, incidentally, is acknowledged as the initiator of the systematic study of monoids and combinatorics on words) and deals with *periodic* properties (where the period $\pi(w)$ of a word w is defined as the minimum length of words admitting w as a factor of some power). Chapter 9 (Christian Choffrut) gives an introduction to the vast subject of *equations in words* (here again the name of Lyndon arises frequently in the discussion). In Chapter 10 Dominique Foata describes how *rearrangements* of words can be used in the enumeration of permutations of finite sequences with certain specified properties (such as a given number of descents or a fixed up-down sequence). The final Chapter 11 (Robert Cori) covers the relationship between *plane trees*, *parenthesis systems* and certain families of words. An interesting aspect of this chapter is the use of the combinatorial properties of Lukaciewicz language to give a purely combinatorial proof of the Lagrange inversion formula of complex analysis!

In reading a book of this nature one is of course prepared to accept a certain amount of "unavoidable irregularity" in the writing due to the varied authorship of the different chapters. In fact the style is surprisingly consistent throughout signifying a remarkable degree of cooperation among the (eleven) writers. The index has one or two omissions and I found just one instance of a term (*biprefix code* on p. 27) being used without having been defined (the natural place would have been in Chapter 1). But on the whole the cross-referencing and indexing are adequate to the reader's needs. There is a number of misprints but most of these are textual rather than symbolic and along with several (typically French) non-standard uses of the English language can be forgiven in an otherwise excellent production.

The book is written lucidly and for the most part so as to be accessible to anyone with a standard mathematical background. It contains a wealth of information and many topics not mentioned in this review are included. Very few results are taken for granted and each chapter ends with a good selection of detailed exercises designed to bring out applications and extensions of the theory. Also contained in each chapter are comprehensive bibliographic and historical notes and discussion on a fair number of open problems to whet the appetite for further investigation. This book is sure to become the standard reference work in a new and potentially fruitful area of mathematics.

P. Fitzpatrick,
Mathematics Department,
University College,
Cork.

"CALCULUS AND ANALYTIC GEOMETRY"

by *Donald W. Trim*

Published by *Addison-Wesley Publishing Company, Inc.* 1983.
Stg £25.45.

ISBN 0-201-16270-9

Academic life is financially secure, but the chances of making a "killing" are few and far between. It is, of course, interesting to speculate how we might get by if we were paid by the theorem; if the mortgage payment next month depended on settling that result you have been trying to prove over the past two years. It might well extend the active mathematical life of many, exposing as a myth the belief that creative mathematics is done by the young. It would certainly make life interesting; it would probably make it shorter too. Most of us are glad that this is not the way things are arranged, and in a society governed by supply and demand we may deduce that we have to be grateful that someone somewhere is giving us the time to prove theorems at all. (Notable exceptions to these observations, indeed most observations, are the U.S.A. and France. The hiring system in the United States has created star and superstar status for certain mathematicians, with corresponding salaries; in France people *are* paid money for proving good theorems!)

One killing however beckons us all: write the perfect calculus text and get rich! But perhaps I am being a little mercenary. With calculus such a stumbling block for so many perhaps the quest for the perfect calculus text is the mathematician's analogue of the quest for the holy grail. Donald W. Trim lays claim to the Siege Perilous. Is it his? Before pronouncing judgement (how much easier it is to write reviews than to write books) let me describe the text.

The first thing that strikes one on picking up the book is its weight! There are over 900 pages and the range of material covered is impressively complete. I will list the chapter headings, the subheadings can be determined by analytic continuation. Chapter 1: Plane analytic geometry and functions; Chapter 2: Limits and continuity; Chapter 3: Differentiation; Chapter 4: Applications of differentiation; Chapter 5: The indefinite integral or antiderivative; Chapter 6: The definite integral; Chapter 7: Applications of the definite integral; Chapter 8: Transcendental functions and their derivatives; Chapter 9: Techniques of integration; Chapter 10: Conic sections, polar co-ordinates, and parametric equations; Chapter 11: Infinite sequences and series; Chapter 12: Vectors and three-dimensional analytic geometry; Chapter 13: Differential calculus of multivariable functions; Chapter 14: Multiple integrals; Chapter 15: Vector calculus; Chapter 16: Differential equations.

The book is very attractively produced, as one might expect from Addison-Wesley, with many useful diagrams, and an extra wide margin down the lefthand side of the pages. (Would the world be a wiser place if this had been the case in Fermat's time?) There are over 4,400 problems, some of which require the use of an electronic calculator, with answers to the even numbered ones in the back. Other available supplements

include a student's manual containing detailed solutions to even numbered exercises, an instructor's manual containing answers and selected solutions to odd numbered exercises (hopefully the hard ones) and a set of transparencies for the more complicated figures in the text. (Thankfully no inflatable lecturers - at least not yet!).

All of this is no doubt much as you might expect and indeed this is true of the text as a whole. It seems to be quite well written, but I did spot some errors and points of contention. On page 42 we learn that a function $f(x)$ has limit L as x approaches a if $f(x)$ can be made arbitrarily close to L by choosing x sufficiently close to a. Although this "definition" is not meant to be precise (there is a "mathematical definition" of limit on page 57) it really is completely misleading, it is "sufficiently close" that one chooses and not x. The other surprising error I spotted occurs on page 444, in exercise 35. "Prove the following result: If $\sum_{n=1}^{\infty} c_n$ converges, then its terms can be grouped in any manner, and the resulting series will be convergent with the same sum as the original series." Presumably the author had something fairly restrictive in mind when he wrote "grouped in any manner", but given the standard results on conditionally convergent series one might have hoped for something a little more precise (or perhaps, even better, nothing at all).

Two other complaints: first why do so many calculus texts discuss differentials? In this book we have a definition "An increment Δx in the independent variable x is denoted by

$$dx = \Delta x$$

and when written as dx is called the differential of x." I am unsure what students make of such stuff, it certainly has me puzzled and moreover undermines any other definition appearing in the text. This approach, to a fairly straightforward topic - linear approximation - is almost certain to cause confusion. Also in the preface to the student the author states that it is surprising that neither Leibniz nor Newton formulated the idea of limit. This seems to me to be an alarming confusion of the logical and psychological, which thankfully is not continued in the text.

How does this book compare with its rivals? There has been no fundamental change in the selection or treatment of material used in calculus texts over the past 20 years, and competitors vary by and large very little. My present favourite of books of this type is Fraleigh's *Calculus with* (rather than *and*) *Analytic Geometry*, also published by Addison-Wesley, and I prefer it to the text under review. It covers almost exactly the same material as the book by Trim, but is more direct and considerably shorter (in content, not pages; the print in Fraleigh's book is larger than that in Trim's). It is to be noted that since neither book deals with complex variables or Fourier series they are really not suitable as recommended texts for engineering students, but more of that later.

And my conclusion? Well I have little doubt that such a book would make a useful addition to any university library. What of the holy grail? Well we all know that *that* quest is part of Arthurian *legend* (which in turn appears to be the English attempt to compensate for the unmistakeable fact that God was a foreigner). Similarly, the perfect calculus text is a fiction; especially so on this side of the Atlantic. For in contrast with the situation in the U.S. one rarely has a number of classes being taught the same material simultaneously, and consequently there is not the same need for some unifying influence. Where possible we all usually prefer to give our own treatment, perhaps gleaned from a number of books, or courses we have attended. Perhaps more importantly (back to money again) one certainly could not recommend a text at this price to a class of students here or in the U.K.

There is one other point, which I think even (or especially) dedicated writers and publishers might lose sight of when considering this as a students' textbook. That is the very completeness of such texts is off-putting. This volume

may well be the lifetime's work of its author; it has the look of the same for any prospective reader. Moreover, many of the interesting applications tend to be, if anything, too interesting and distracting, and the exercises suffer a little from the same symptoms, with too few trivial ones. The author says in his preface to the student that "The key word in our approach to calculus is *think*." I hope he does not claim any originality here. But nonetheless thinking can be a rather elusive and overestimated quantity in the learning process. A selected and condensed core of material, to be learnt by rote, and a number of mechanically (and hopefully quickly boring) examples may be great aids to understanding. Indeed, here we may have the main reason why calculus causes so many problems, and why calculus texts are only ever a minor aid to its understanding. With calculus, as with any other worthwhile topic, one has to be willing to soldier on in a fog for a considerable time before (hopefully) sunshine filters in, and the promise of the joys of understanding is usually not enough for (at least normal) students. So how do we do it? How do we get them to soldier on? At school by intimidation, but later? I'm not quite sure, but people really are more interesting than mathematics, so don't lose too many nights sleep worrying about those inflatable lecturers.

J.W. Bruce,
Mathematics Department,
University College,
Cork.