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NEWS FOR COMPLEX VARIABLES TEACHERS

Aongus Ó Cairbre and Anthony G. O'Farrell

It is good pedagogical practice to mix solid applications (inside or outside mathematics) with the development of general theory [3]. This is routinely done in general courses, but it is just as important in honours courses, because otherwise the students may get the wrong idea of what mathematics is and how it is done. Mathematics is best done with a specific problem in mind.

People who agree with this point of view will be interested to learn that two major applications of complex function theory have recently been simplified to the point where both can now be presented to average honours undergraduates. Hitherto they were, realistically speaking, first year postgraduate material. The results are the big Picard theorem and the prime number theorem.

The original proof of Picard's theorem, using the elliptic modular function and monodromy, remains firmly at postgraduate level. Of course it was undergraduate material long ago, when it was acceptable to be vague about topological problems. Until last year, the proof normally used was basically that in Landau's "Neuere Ergebnisse", via the theorems of Bloch and Schottky. The new proof is a simplification of this latter proof. It gets the result in one page after Schottky's theorem. The entire proof, assuming the maximum principle, Rouché's theorem, and a knowledge of the logarithm and complex powers, may be presented in two lectures. The new idea is due to Bridges, Calder, Julian, Mines, and Richman, and is explained in [2]. Curiously enough, they found this simpler proof, not because they were trying to, but because they sought a constructive proof, i.e. one not using the apparatus of normal families.

The new proof of the prime number theorem is due to Newman [6]. Until it appeared, the simplest proof was that in Heins' "Topics". The latter proof involved the Riemann-Lebesgue lemma and many technical convergence details. Newman actually offered two proofs. He started by giving an ingenious proof of a Tauberian theorem of Ingham. He observed that Landau's equivalent form of the prime number theorem follows at once. He went on to give the details of another proof, based on the fact that the existence of the limit

$$= \lim_{n\to\infty} \sum_{\substack{p<n \\ p \text{ prime}}} \frac{\log p}{p} - \log n$$

implies the prime number theorem. Korevaar [4] has produced a variation on Newman's method. Korevaar's version is shorter than either of Newman's proofs. It can be presented in three or four lectures, including the basic facts about the Riemann zeta function. Newman's first proof, with all the details included, takes about five lectures. Some of the details are sufficiently straightforward to be left to students. We are inclined to favour Newman's first proof, even though it takes more time, because the proof of the sufficiency of Landau's equivalent form depends upon two gems of number theory, namely the Möbius inversion formula and Dirichlet's estimate

$$d(1) + d(2) + \ldots + d(n) = n \log n + (2\gamma - 1)n + O(\sqrt{n})$$

where $d(n)$ is the number of divisors of n, and γ is Euler's constant. The proof of Landau's equivalent form is in [5]. A more recent reference is [1], which contains a complete account, with all the details, of Newman's first proof.

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