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Riccardo Benedetti: Lectures on Differential Topology, AMS, 2021. ISBN: 978-1-4704-6674-9, USD 85.00, 425 pp.

REVIEWED BY BENJAMIN MCKAY

Both the quantity and quality of textbooks decays, on average, with the level at which they are set, leaving us with so many excellent textbooks in calculus and linear algebra, and so few bridging the gap between basic graduate topics and current research. This book makes a daring attempt to help the student get a big picture of differential topology. The author develops the central pillars of differential topology on compact manifolds: transversality, bordism, cobordism, characteristic numbers, 3-manifolds and 4-manifolds, starting by laying out some basic definitions and theorems in the concrete context of open sets in Euclidean space, and then for embedded submanifolds of Euclidean space. This book is written for graduate students in mathematics without prior study of algebraic topology; there are many excellent books in this area (we mention a few below), but still a wide gap between the books and research. The author does not mention whether he expects the reader to have prior familiarity with differential geometry, but the pace would be too quick for a first encounter with manifolds.

Benedetti's approach is quite concrete, emphasising cut and paste. The classical approach to characteristic classes of Pontryagin looks at the zeroes of generic sections of vector bundles, varying with the choice of section, following the algebraic geometer's picture of a pencil of curves on a surface. This picture reveals that these sections all have the same homology class, which is thus a topological invariant of the vector bundle itself. The author explains that all vector bundles are pulled back from the tautological vector bundles on Grassmannians, hence from the tautological vector bundle on the infinite Grassmannian, and therefore the vector bundles are classified by the homotopy classes of maps to the infinite Grassmannian.

The author provides the bulk of Whitney's proof of his embedding and immersion theorems, including the cancellations of oppositely signed double points using the Whitney trick. His insistence on working only with compact manifolds makes the existence of a Euclidean embedding almost obvious, but he chooses to follow the path of Whitney, which gives the proof for noncompact manifolds and also gives information about embedding dimensions. Again, the approach is through transversality arguments and through elementary cut and paste. He approaches Morse theory, in much the style of Milnor's book, emphasising handle decompositions and discussing handle sliding. Then he works through bordism and cobordism invariants, some theory of 3-manifolds and 4-manifolds including the Arf invariant.

At some points, the book runs very quickly through subtle points. The reader might like to have C.T.C. Wall's book [8] on hand. Compared to the textbooks of Adachi [1], Bröcker and Jänich [2], Hirsch [3], Milnor [4, 5], Munkres [6], Shastri [7], Wall [8], or Wallace [9], this book emphasises low dimensional topology more, and proceeds closer to current research, making more frequent use of references to proofs so as to skip some of the complicated bits of the proofs. The index is unfortunately brief.

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References

[1] M. Adachi: *Embeddings and immersions*, Translated from the 1984 Japanese original by Kiki Hudson. Translations of Mathematical Monographs, 124. American Mathematical Society, Providence, RI, 1993.

[2] T. Bröcker and K. Jänich, *Introduction to differential topology*, Translated from the German by C. B. Thomas and M. J. Thomas. Cambridge University Press, Cambridge-New York, 1982.

[3] M. Hirsch, *Differential topology*, Corrected reprint of the 1976 original. Graduate Texts in Mathematics, 33. Springer-Verlag, New York, 1994.

[4] J. Milnor, *Morse theory*, Annals of Mathematics Studies, No. 51 Princeton University Press, Princeton, N.J. 1963

[5] J. Milnor, *Topology from the differentiable viewpoint*, Princeton Landmarks in Mathematics. Princeton University Press, Princeton, NJ, 1997.

[6] J. R. Munkres, *Elementary differential topology*, Annals of Mathematics Studies, No. 54 Princeton University Press, Princeton, N.J. 1966

[7] A. R. Shastri: *Elements of differential topology*, CRC Press, Boca Raton, FL, 2011.

[8] C. T. C. Wall, *Differential topology*, Cambridge Studies in Advanced Mathematics, 156. Cambridge University Press, Cambridge, 2016.

[9] A. Wallace, *Differential topology. First steps*, Reprint of the 1979 (third) printing of the 1968 original. Dover Publications, Inc., Mineola, NY, 2006.

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