Irish Math. Soc. Bulletin
Number 92, Winter 2023, 62–64
ISSN 0791-5578

Ron McCartney: A Gentle Path into General Topology, Bookboon, 2022.
ISBN:978-87-403-4370-0, GBP 00.00, 256 pp.

REVIEWED BY AISLING MCCLUSKEY

This book, e-published and currently freely available by Bookboon, comes in three parts encompassing a total of eight chapters of introductory material. Its content emerged from an undergraduate course developed in Asia by the author over an eight-year period. The author indicates a supplementary *What you need to know for Chapter* n (WYNTK n) feature that is available on the Bookboon website. He provides this as a useful gauge for the reader to assess their level of preparedness for a course based on the book. Forewarned is forearmed!

The book is pitched at the level of undergraduates who are meeting general topology for the first time, typically within the last two years of their degree programme. It is written in a style that recognises and embraces the step-up required in handling abstraction. The book is aptly named in that regard as the author scaffolds the learning in a gentle, well-paced manner borne of long experience in careful, thoughtful teaching.

Each of the book's three parts is prefaced by a call for the reader to reinforce the basic skills needed to fare well in the course – these are of course the usual suspects such as basic set theory notation including functions and their inverses, limits in $\mathbb{R}$, manipulation of absolute value inequalities, intervals in $\mathbb{R}$, and knowledge of proof methods. The cognitive and pedagogical piece in terms of advising how the students should undertake the 'gentle path' ahead is also comprehensively captured and repeated thrice over in each part's Introduction, with mild customisation towards the book part it heralds. As mathematicians and educators, this advice is ingrained in us – but we cannot say it often enough to our students. It is useful to have this core advice integrated and reinforced throughout. A further pedagogical aide is the peppering of reflections throughout the book under the guise of *Think about ...*.

Part 1 introduces metric spaces (Chapter 1) and their open sets (Chapter 2). Chapter 1 is chock-full of examples, non-examples and exercises which serve to strengthen an understanding of the cornerstone definition of a metric space. One or two of the examples are a little too informal or too vague for my taste (for example, the set S in Example 1.3 is introduced as 'quite a large set of people'; Example 1.6 refers to the imagery of mangoes in the interpretation of adding a new point to the set $\mathbb{N}$!) but the slow, worked-out solutions provided will be invaluable to the conscientious learner. Interestingly the taxicab metric (or Manhattan metric) is called the postal metric in Exercise 1.3 – this may be a contextual decision. Chapter 2 then introduces the concept of an open ball leading to an open set. Enroute the term *neighbourhood* is used without prior definition – so there is a sequencing issue here. (In fact, neighbourhood in a topological space is defined later in Chapter 3.) An open ball of radius r is referred to as an r-open ball which is not a turn of phrase I have come across before. There may be some potential here for confusion given how open sets are subsequently introduced

Received on 09-06-2023; revised 04-07-2023.
DOI:10.33232/BIMS.0092.62.64.

as 'd-open sets' where d is the given metric. Part 1 ends appropriately by setting out the properties of open sets that signpost the imminent arrival of a topological space.

Part 2 right on cue proceeds with an introduction to a topology (Chapter 3) and to continuity (Chapter 4). It wends its way through a variety of standard 'training' topologies and the occasional non-topology, presenting a strategy for students to check when a collection of sets forms a topology. Interestingly the author includes a section (3.2) on the ordering by inclusion of topologies on a given set. Section 3.3 raises the important and classic question concerning whether a given topology arises from a metric. This is a nice prelude to the introduction of the Hausdorff property. Further sections are devoted to the concepts of basis for a topology, subspaces, closed sets and closures of sets. Chapter 3 concludes with an investigative nod towards the equivalent way of defining a topology via neighbourhoods and of a characterisation of closed sets via sequential convergence. Chapter 4 takes on the concept of continuity, building up from its meaning for real-valued functions on $\mathbb{R}$ through metric spaces to topological spaces and paying close attention to detail as it does so. It is tried out on various functions and the role of a basis is usefully brought into play in the process. Section 4.3 on homeomorphisms is particularly important in this chapter. Section 4.5 on product spaces is also important but it is let down somewhat by some typesetting errors (for example, pages 91 and 94). While the author does indicate that Section 4.7 on quotient spaces borrows heavily from Jänich's *Topology* (Springer), the key definition of a quotient map on page 96 is unclear (a_k is not defined). Part 2 is quite substantial, running to 101 pages, but there are standalone sections that can be omitted on first reading/study under the guidance of an instructor.

Finally, Part 3 clips along at a steady pace with four chapters on compactness, connectedness, completeness and separation axioms respectively. The treatment of compactness is nice, involving its interaction with Hausdorffness, its behaviour under continuity and on taking subspaces and referencing other authors for investigation of more advanced topics such as compactifications. Connectedness follows a similar approach with some nice discussion on components and a lovely selection of examples to illustrate non-homoeomorphic spaces. Completeness of necessity requires some preliminary work on sequences and subsequences but quickly ramps up to consider the property alongside compactness. A reasonably extensive investigation section on topics for further study is also provided. The final section on separation axioms then comes somewhat as light relief but it nonetheless does get to a discussion of normality and its waywardness (by comparison with the weaker axioms). The cohesion of compact Hausdorff in this section is pleasing. Appropriately the investigative add-on for this section covers issues of metrizability and products and subspaces of normal spaces.

As a more general comment, the book is not prepared in LateX and so the mathematical notation and general formatting is non-standard which can detract from the overall presentation. For example, $\mathbb{R}^n$ is rendered as R^N in a capitalised title. In Part 1, the title in 2.3 featuring 'r-open balls in $\mathbb{R}$' is rendered "R-open balls in R". The inevitable typos, the bane of every author, also carry an impact (for example in Part 3, sequence notation depicted as xn on page 50, two different fonts for ϵ on page 56); use of 'equivalent' in Section 7.5.2 (page 65) where 'compatible' might be a better choice given the significance of the word in an earlier discussion on equivalent metrics. Other idiosyncracies occur such as the reference to a circle when the intention is the entire disk (so circle and its interior) in the plane. Professionals can readily pass over these, understanding what is meant - but they can trip up the beginner.

Fundamentally the book constitutes a thoughtful and student-centred companion to similar undergraduate texts in the market and will be a useful resource for student and instructor alike.

Aisling McCluskey was educated at Queen's University Belfast and is a Personal Professor in Mathematics at the University of Galway. She is currently a governor at the University and Head of the School of Mathematical and Statistical Sciences there. Her research interests lie within analytic topology fuelled by a lifetime interest in and commitment to undergraduate mathematics education.

School of Mathematical and Statistical Sciences, University of Galway
E-mail address: aisling.mccluskey@universityofgalway.ie