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J. Cruickshank et al.: Irish Mathematical Olympiad Manual, Logic Press, 2023.
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REVIEWED BY JESSICA SEARANCKE

One of the first things I noticed about the the Irish Mathematical Olympiad Manual is the way it begins: with general problem-solving advice, quickly followed by a summary of the basic knowledge any reader should already have from the Junior Certificate. I have to say it bluntly: the problem-solving advice is really good. Stuff that should be straightforward, and that makes a massive difference to your abilities, and yet so many students don't know it: Spread out your work because paper is cheap. Understand the target fully, and find proxy targets to help solve it. Lots of these tips I use already, some of them I don't - but regardless, I've never seen them written down in this way before. Lots of people view problem-solving as something you either can do, or you can't do, in a direct relationship with your IQ score. By putting problem-solving skills down to intelligence alone, people tend to ignore the core skills that make the difference between someone who's confused and gives up, and someone who's confused but figures it out. This book puts it differently, listing lots of the skills that make this difference.

My only criticism of this section of the book is its understandability. I could immediately understand and relate to the skills that I currently use, but it was less easy to fully grasp how to use the unfamiliar skills in a real problem. Inserting an example or two of how the techniques could be used, following this section (or each point individually), would make a real difference to how easily students can apply the advice given. The problems at the end of the chapter go some way toward this goal, but they lack worked solutions, or specific links to the tips. Worked solutions would demonstrate the advice, and would change the way students work through problems.

The next section of the book reviews the baseline knowledge that is needed before using the manual. The way in which it concisely lists rules and facts is ideal for this kind of book, since readers are likely to already be familiar with such information, but could do with a reminder, or a point for future reference. For me, such rules are far too easy to forget, but can also be quickly refreshed into the working memory. Similarly, the review of basic trigonometry clearly demonstrates the progression from this baseline knowledge to the far more complex use of these functions in later chapters (including De Moivre's theorem). Such progression illustrates the versatility of the manual for a range of abilities of reader, since it spans several years of schooling. Unfortunately page 6 appears to contain an error on its first line ('concyclic!points'). I expect this to change in future editions. *(Editor's Note: this is corrected in the direct-sales edition).*

Past this point, the book begins to review or teach (depending on your position) a range of different concepts. Having already studied maths at a high pre-university level, I am familiar with all of the initial concepts, I have encountered many of the intermediary concepts, and lots of the most complex concepts are completely new to me. The entire book is formatted in black and white text, which is spread out clearly and logically. This separates the manual from a revision guide: it is a workbook, to be

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worked through in order, if possible, to gradually develop further mathematical skills. I believe this book would be ideal, not only to prepare for a Mathematical Olympiad, but also to develop essential skills prior to studying maths or other related subjects at university. The skills developed in this book cannot be taught from a one-off YouTube video or web search, but can be gained over time by working through a manual such as this one. The inclusion of regular practice problems and examples supports this gradual skill development.

To criticise the main body of the book, I would like to first address the use of proofs to teach new topics. They are heavily used to introduce new ideas, from the sine and cosine rules, all the way to Ceva's theorem. For me, I completely endorse the use of proofs to do this, however I feel that these could sometimes be better explained. For example, many proofs might be useful in understanding a concept, but the application of this knowledge is more important. In some cases, the proof dominates the explanation of a concept. In other cases, it is unclear whether the authors intend the resulting fact to be learnt and used by rote, or whether they are simply introducing the reader to a way of relating different facts in a problem. Further worked problems would help solve this issue by demonstrating how the facts are intended to be used, and how they are most likely to come up in a real Olympiad question.

In order to better explain the issue I have just raised, I will give some examples: The beginning of Section 6.2 (Combinatorics and Binomial Coefficients) is very well explained - it begins by addressing the meaning of the factorial symbol, explains logically how numbers of combinations and permutations are worked out, and gives some examples to demonstrate this point. Similarly, Section 10.2 (Some Theorems about Triangles) addresses each theorem individually, accompanied by several example questions for each. However, also in Section 10.2, some of the exercises are heavily proof-dominated (such as for Ptolemy's theorem). The reader will have little understanding of how to apply their new knowledge, only how to prove it. Even if Olympiad questions do have a tendency to focus on proof, understanding can be significantly enhanced by candidates also knowing how to apply the concept.

On page 25, a complex proof is given involving the semiperimeter, however no diagram is included. As a result, it is unclear to a reader what 'semiperimeter' refers to (if this is explained elsewhere in the book, a page reference should be given). The meaning of a, b, c, A, B and C - all letters which are used in the proof - is also unclear. As well as this, on page 29, the relationship between an inscribed circle and the surrounding triangle is described. In my experience, this is extremely useful to recognise in Olympiad-style questions. However the formula given is one that I would be unlikely to memorise pre-exam. Do the authors simply wish to introduce the reader to this style of comparison, or do they actually recommend learning the formula? The answer to this question should be explicit. Also, if the former is true, I think there could be more effective ways of introducing this concept. My final point to address is the index, which is highly comprehensive. I would only like to question the inclusion of specific angles at the beginning of the index, for example $15°$ and $270°$, which seems unnecessary and of little use.

Overall, I believe the book is almost ideal for all those studying maths pre-university, including those preparing for Mathematical Olympiads. It is effective in developing problem-solving skills, including those involved with proof. My criticisms widely revolve around the necessity for further examples to demonstrate some of the concepts taught. I would highly recommend this book for able students wishing to push their studies in mathematics further in their final one or two years of school, and those developing essential mathematical skills prior to entering university.

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