IMS Annual Scientific Meeting 2023
University of Limerick and Mary Immaculate College
31st August - 1st September

This year's annual meeting of the Society took place on 31st August and 1st September at the University of Limerick. The meeting was organised collaboratively by the Department of Mathematics and Statistics and the Mathematics Application Consortium for Science and Industry (MACSI) at the University of Limerick (UL) and the Department of Mathematics and Computer Studies at Mary Immaculate College (MIC). The organising committee consisted of Romina Gaburro (co-chair), Eugene Gath, Natalia Kopteva, Kevin Moroney and Clifford Nolan at UL and Ronan Flatley and Derek Kitson (co-chair) at MIC. This was the 36th annual scientific meeting of the Society and the meeting attracted over 70 participants. The event opened with a welcoming address from Dr Marie Connolly, Director of Human Rights, Equality, Diversity and Inclusion at UL. The schedule of talks featured over a dozen experts from across Ireland and the United Kingdom (a complete book of abstracts follows this report). The invited speakers were:

- Norma Bargary, University of Limerick,
  Functional data analysis for sports analytics;
- Patrick Browne, Technological University of the Shannon,
  Segre's theorem on ovals in Desarguesian projective planes;
- John Butler, Technological University Dublin,
  Mathematical Modelling of Multisensory Neuronal Processing and Behavioural Responses;
- Julie Crowley, Munster Technological University,
  Exploring the relationship between Mathematics and Emotions;
- James Cruickshank, University of Galway,
  Rigidity of bar and joint frameworks: polyhedra and beyond;
- Patrick Farrell, University of Oxford,
  Computing multiple solutions of nonlinear partial differential equations;
- Ivan Graham, University of Bath,
  Convergence of iterative solvers for the Helmholtz equation at high frequency;
- Emma Greenbank, University of Limerick,
  Volcanism to Batteries; modelling fluid flow in porous media;
- Thomas Huettemann, Queen's University Belfast,
  Some remarks on the "fundamental theorem" in algebraic K-theory;
- Cónall Kelly, University College Cork,
  Adaptive numerical methods for stochastic jump differential equations;
- Bernd Kreussler, Mary Immaculate College,
  On twistor spaces from an algebraic geometry perspective;
- Myrto Manolaki, University College Dublin,
  Holomorphic functions with chaotic behaviour;
- Katrin Wendland, Trinity College Dublin,
  Some quartic K3 surfaces.

The programme included a poster session for postgraduate students and early career researchers. A €100 prize for best poster by a postgraduate student was sponsored by SIAM UKIE. This sponsorship was a new feature of the meeting and the winner was David McMahon (UL) for his poster titled *Microlocal Analysis of Multistatic Radar Imaging.* The poster presentations were:

- Milton Assunção, University of Limerick,
  Dissolution of drug particles subject to natural convection;
- Jason Curran, University of Limerick,
  Stability and Reconstructions for Anisotropic Diffuse Optical Tomography;
- Niall Donlon, University of Limerick,
  Stable Reconstruction of a special type of anisotropic conductivity;
- Abdul Fatah, Atlantic Technological University, Galway,
  Quantum Error Correction using Quantum Latin squares;
- Oisín Flynn-Connolly, Université Sorbonne Paris Nord,
  The geometry of iterated suspensions;
- Seán Kelly, University of Limerick,
  Pointwise-in-time error bounds for a fractional-derivative parabolic problem on quasi-graded meshes;
- David McMahon, University of Limerick,
  Microlocal Analysis of Multistatic Radar Imaging;
- Shraddha Naidu, University of Limerick,
  Inclusion Size detection with Deep Learning and electrical impedance tomography;
- Maged Shaban, Technological University Dublin,
  Influence of Ceramic Suspensions on Micro Stereolithography Printing Modeling and Simulation.

Another new feature of the meeting was to invite short talks and poster presentations from the most recent winners of the Royal Irish Academy Hamilton Prize in Mathematics. The contributors were:

- Tiernan Brosnan, University of Limerick,
  Real-Time High-Frequency Radar Imaging (Short talk);
- James Hayes, University of Galway,
  Contact Graphs (Short talk);
- Ryan McGowan, Trinity College Dublin,
  Classifying Domains in $\mathbb{C}^n$ *through the Study of Metrics (Poster)*;
- Kituru Ndee, Technological University Dublin,
  Fluid Dynamics: Slow Viscous Flows (Poster).

The Annual General Meeting took place on the Friday at which 30 new members were welcomed to the Society. The webpage for the meeting is archived at:
`https://www.ul.ie/scieng/schools-and-departments/department-mathematics-and-statistics/36th-annual-meeting-of-the`

The organisers would like to express their gratitude for financial support provided by the Irish Mathematical Society, Mary Immaculate College (Department of Mathematics and Computer Studies), the University of Limerick (Department of Mathematics and Statistics/MACSI, Faculty of Science and Engineering, UL President, CONFIRM, SSPC), IBEC and SIAM UKIE. The organisers would also like to thank all who assisted with the organisation of the meeting, particularly the Royal Irish Academy and Dana Mackey (TU Dublin). Finally, the organisers would like to thank all who contributed to the meeting; the invited speakers, those who presented posters, and all who attended for creating an informative and convivial meeting.

Report by Romina Gaburro (UL) & Derek Kitson (MIC)
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Figure 1. Participants at the 2023 IMS Annual Meeting.
(Photo courtesy of Thomas Huettemann (QUB))

BOOK OF ABSTRACTS:

Functional data analysis for sports analytics

Norma Bargary
University of Limerick

Functional data analysis (FDA) is a statistical methodology that is suitable for modelling high-dimensional data collected over some continuum (typically time). FDA is particularly suitable for the modelling and analysis of data that are measured continuously, more recently via state-of-the-art sensor technologies. This talk will outline the role of FDA with applications to data collected in a variety of sports settings, and discuss the methodological challenges associated with the statistical modelling of these modern human movement datasets.

Segre's theorem on ovals in Desarguesian projective planes

Patrick Browne
Technological University of the Shannon

Segre's theorem on ovals in projective spaces is an ingenious result from the mid-twentieth century which requires surprisingly little background to prove. In this brief talk we give a self contained proof of Segre's theorem. This is accessible to most yet showcases some minor improvements to Segre's proof that allow for results in shorter time and simpler computations than the original.

Mathematical Modelling of Multisensory Neuronal Processing and Behavioural Responses

John S. Butler

Technological University Dublin

Efficient navigation through the world heavily relies on the seamless integration of signals from multiple sensory modalities within the brain. Previous behavioural studies have suggested the existence of a winner-take-all sensory response mechanism as well as an optimal combination of sensory signals during multisensory processing. Conversely, evidence indicates that maladaptive multisensory processing could serve as an indicator of older adults' susceptibility to falls when compared to age-matched healthy controls.

While the work of Wong & Wang (2006) has been influential in modelling sensory decision-making, most research to date has focused on unisensory tasks. To address this gap, I will present an extension of their reduced two-variable model, designed to simulate both unisensory and multisensory neuronal processing and behavioural responses. This model is built upon ordinary differential equations, driven by biological data, enabling the investigation of audio-visual speeded reaction-time tasks and visual-vestibular decision making.

In this talk, I will illustrate how the extended model successfully re-creates and tests previously observed behavioural findings. Additionally, our model provides novel insights into the proportion of unisensory and multisensory neurons required for optimal multisensory integration. This work has been done in collaboration with Rebecca M. Brady (IRC funded PhD student).

Exploring the relationship between Mathematics and Emotions

Julie Crowley

Munster Technological University

Do you think it is reasonable to have the words Mathematics and emotions in the same sentence? Is Mathematics emotional? In this talk we investigate the relationship between Mathematics and emotions using a longitudinal case study. We also explore the possible relevance of this topic to a mathematics lecturer.

Rigidity of bar and joint frameworks: polyhedra and beyond

James Cruickshank

University of Galway

Bar and joint frameworks are objects that arise naturally in many mathematical contexts, from applications to mechanical and structural engineering, to protein folding, to commutative algebra, recreational mathematics and elsewhere. Their mathematical theory is a rich and currently very active area of research. In this talk I will survey some old and new results relating polyhedra and the rigidity theory of frameworks. The talk will be accessible to a general mathematical audience, by which I mean anyone who knows what a graph is and what a polyhedron is.

Computing multiple solutions of nonlinear partial differential equations

Patrick Farrell

Oxford University

Nonlinear problems may support multiple solutions, and these multiple solutions are typically very important for the application at hand. Examples include the buckling and snapping of structures, bistable devices in computer memory and displays, high- and low-confinement regimes in tokamak fusion reactors, multiple local minima of nonconvex optimisation problems, multiple Nash equilibria in games, and so on. Despite their mathematical interest and physical importance, the calculation of multiple solutions is not routinely carried out by practitioners, due to a lack of good algorithms. In this lecture I will present an elegant algorithm, based on Newton's method, for computing multiple solutions of nonlinear partial differential equations.

Convergence of iterative solvers for the Helmholtz equation at high frequency

Ivan Graham

University of Bath

Many interesting applications require the solution of the linear wave equation in heterogeneous media and/or complicated geometry - for example forward and inverse scattering problems in acoustics or electromagnetics. When the data oscillates on a restricted range of frequencies, application of Fourier transform can remove the time dependence, leading to the Helmholtz equation - an indefinite linear second order elliptic PDE. However (at high frequency), this equation is non-coercive (in standard settings) and has highly oscillatory solutions.

To compute solutions, fine discretizations (finite element methods) are required, resulting in (sparse) systems of linear equations with millions of unknowns and highly indefinite system matrices. Because of the system size and structure, application of modern direct methods (i.e., clever variants of Gaussian elimination) are problematic, so there is great interest in finding iterative methods which compute the solution via a sequence of 'local' approximations, (so-called 'domain decomposition' methods). Such methods are well-suited to implementation on modern parallel hardware and the search for fast convergent iterative methods for the discrete wave equation (guaranteed by theory) is thus a very active current research topic in numerical analysis/scientific computing.

In the low-frequency case, the PDE behaves like the Poisson equation, the linear systems are symmetric positive definite, and many 'optimal' methods (the most famous being 'multigrid') are available for fast iterative solution. However these methods generally fail at high-frequency.

In the talk I'll present some theory of the Helmholtz equation and its discretization, and a simple iterative procedure which forms the basis of several successful practical large-scale solvers. Then I'll present some recent theory which explains the convergence properties of this method.

The techniques of analysis involve combining the theory of the Helmholtz equation at high frequency with numerical analysis of finite element and domain decomposition methods.

The work is joint with Shihua Gong (Chinese University of Hong Kong Shenzhen) and Euan Spence (Bath). Some results are also joint with Martin Gander (Geneva) and David Lafontaine (Toulouse).

Volcanism to Batteries; modelling fluid flow in porous media

Emma Greenbank

University of Limerick

During this talk I will be discussing two different modelling projects arising from my PhD and Post-doctoral work. The first application I will be focusing on is from the field of volcanism. The eruptions that are of interest to this research are those that occur through crater lakes or shallow sea water. These eruptions are often some of the most dangerous in the world as they can cause tsunamis, lahars and base surges, but the phenomenon of interest for this research is that of the Surtseyan ejecta. Surtseyan ejecta are balls of lava containing an entrained material. They occur when a slurry of previously erupted material and water washes back into the volcanic vent. This slurry is incorporated into the magma and ejected, from the volcano, inside a ball of lava. Despite the formation of steam and anticipated subsequent high pressures inside these ejecta, many survive to land without exploding. The aim of this research was to explain the ejecta survival by describing the coupled evolution of pressure and temperature due to the flashing of liquid to vapour within a Surtseyan ejecta while it is in flight. Analysis of the model provides a criterion for fragmentation of the ejecta due to steam pressure build-up, and predicts that if diffusive steam flow through the porous ejecta is sufficiently rapid, the bomb will survive the flight intact. This criterion explicitly relates fragmentation to ejecta properties, and describes how a Surtseyan ejecta can survive in flight despite containing flashing liquid water, contributing to an ongoing discussion in volcanology about the origins of the inclusions found inside bombs. The remainder of the talk will be focusing on the modelling of a battery anode design introduced in 2019. This anode is composed of an array of copper nanowires, coated with Li-carrying copper silicide and surrounded by Li-alloying electrolyte. During the charging of the anode the bed of nanowires are deformed into a porous structure. This design has shown promise in alleviating the issues cause by silicon in lithium ion batteries. One major challenge of silicon is the extreme volumetric change of silicon during lithiation. The stresses resulting from this swelling can cause degradation and failure of the battery. In this research numerical solutions of the homogenised problem are used to predict the transport of lithium through the anode.

Some remarks on the "fundamental theorem" in algebraic K-theory

Thomas Huettemann

Queen's University Belfast

Two well-known K-theoretical results, the "fundamental theorem" and the computation of the K-theory of the projective line, are traditionally phrased in terms of Laurent polynomial rings. I will indicate how to adapt the statements and proofs to a much larger class of rings, containing, for example, skew Laurent polynomial rings and Leavitt paths algebras of (nice) graphs. In the first part of the talk I will introduce the classical lower K-groups and present some motivation for studying them, before formulating the general results (which are valid for "higher" K-theory as well).

Adaptive numerical methods for stochastic jump differential equations

Cónall Kelly

University College Cork

Stochastic differential equations (SDEs) are used to model the evolution of real-world phenomena subject to random noise and uncertainty. Consider, for example, asset prices or stochastic interest rates in finance, models of ecological systems with complex interaction between species or models of chemical reactions in biological cells. The random noise may act as a diffusion, for example reflecting market volatility, or as a jump process, for example when an ecosystem is subjected to random external shocks. For most nonlinear SDE models there is no closed-form solution and typically numerical methods are used by modellers. However, standard schemes based on solving to a final time using a uniform step size are not applicable for highly nonlinear systems and the methods that do exist are often inefficient.

In this talk we discuss the use of adaptive timestepping for SDEs driven by both a standard Brownian motion and a Poisson jump process. In the absence of jumps, we can ensure strong convergence of explicit schemes, under conditions where they fail to converge over a uniform mesh, by adjusting the timestep in response to the local behaviour of observed trajectories. We will motivate and characterise these strategies and show how they extend to include the jump case. Some implementation issues will be illustrated via a stochastic model of telomere shortening. This is joint work with Gabriel Lord (Radboud University, The Netherlands) and Fandi Sun (Heriott-Watt University, Edinburgh, UK).

On twistor spaces from an algebraic geometry perspective

Bernd Kreussler

Mary Immaculate College

The twistor spaces considered in this talk are 3-dimensional complex manifolds that can be described by polynomials in a concrete manner. I will report about joint work in progress with Jan Stevens in which we answer an interesting open question about deformations of such spaces. The history of the topic and the most important terms and ideas will be explained in a way that is accessible to the non-specialist.

Holomorphic functions with chaotic behaviour

Myrto Manolaki

University College Dublin

This talk is concerned with holomorphic functions which, under a certain countable process, can approximate every plausible function. It turns out that this behaviour, which seems quite pathological, is generic. After presenting several classical examples of this phenomenon, I will focus on some specific cases which have been recently investigated. As we will see, the boundary behaviour of the corresponding holomorphic functions is extremely chaotic.

Some quartic K3 surfaces

Katrin Wendland
Trinity College Dublin

We give an introduction to the geometry of certain complex surfaces, known as K3 surfaces, where we focus on a class of examples given by quartic equations. These special quartic K3 surfaces exhibit a number of beautiful geometric features which also open the door for applications in number theory and in conformal quantum field theory. We will highlight some of these applications in the talk.