

An Interview with Professor Anthony G. O’Farrell

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ABSTRACT. This article consists of an interview with Prof. Anthony G. O’Farrell charting some of his mathematical and life’s journey over the past eight decades. It begins with his mathematical awakening in primary school in 1950s Ireland, through secondary school in Templemore and Drimnagh Castle, his undergraduate days in UCD via a Met Cadetship with the Met Office, and his postgraduate work in the US after receiving the NUI Travelling Studentship. It explores his return to take the first Professorship to ever be offered to a lay person in St. Patrick’s College, Maynooth at the age of just 27 years. It touches on his subsequent 50 plus years of research in mathematics, in which he remains active to the current day, and his overall contribution to Irish mathematical life.

1. INTRODUCTION

I recorded an informal interview with Tony O’Farrell at his home on October 4th 2024. I edited the transcript of this interview, which Tony then reviewed and edited. I subsequently edited the final version of the document.

PM: *The first question is to ask you about your early schooling, in particular, when did you realise that you were interested in mathematics?*

AOF: Well I remember a few things. I started school in Roscrea for a few months and then we moved over to Templemore and I went to the Convent of Mercy in Templemore. I remember when this nun – now I don’t remember her name – showed us how you could add numbers expressed in decimal form – you could add 21 and 33 by just adding the digits up. The light bulb went off when I saw that. That was a non-obvious kind of a trick, which was going to work for adding these big numbers together, and I thought that is pretty good, and it is!

Actually the algorithms that we learned for adding and multiplying decimal numbers were a major advance – they are old but they are effective. I used to do them for fun. Once I learned about multiplication, I would write down a random 10 digit number and another random 10 digit number and then multiply them together so I would have a big long page like this of all the stuff set out. I enjoyed those kind of things.

The next piece of mathematics that I remember that was interesting was the quadratic equation. In 3rd or 4th class we used to graph quadratics – we had graph paper – we had copies with squares on them. You would plot some points on the quadratic and you joined them up by hand.

I remember Rory Geoghegan telling me – his big brother was in 5th or 6th class – that in 5th or 6th class they did something else. They didn’t just plot those things but they explained why they looked like that, so that struck me as an interesting thought. Then there was one day – it was either in 5th or 6th – it is funny because I talked to

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FIGURE 1. Tony O'Farrell

Brother Skehan about this afterwards in more recent times. Brother Skehan was only about fifteen years older than me – but to me he was a very old man to look at. He was teaching us everything, of course, because it was primary school.

One day he explained how you could start with the quadratic equation $ax^2+bx+c = 0$, take the c over to the other side, complete the square so that you got

$$\left(x + \frac{b}{2a}\right)^2$$

equal to something and then take the square roots of the two sides and subtract the $\frac{b}{2a}$ from the other side and you get this formula and it went step by step down the board. I just thought, God, this is an entirely different level of operation, this is something interesting.

I was talking to him years afterwards and I told him this. I said that made a big impression on me and he said: “You know, I remember that too, because it was one of those long days – and I was just feeling the way you sometimes feel, and so I thought, ah, I will just do it!”; so he just did this and he said: “I could tell that you were paying attention – the rest of them were all over the place”. The usual thing – it was a double class because there were three school rooms in the school, one for second class – there was no first class – you went straight into 2nd class – one for 3rd and 4th and one for 5th and 6th. This was 5th and 6th so there would have been maybe 70 kids across the room in the two classes.

So he was telling one group about this – probably the 6th – but I remember the event, the logic of the thing. You were just doing this and it was like a steel trap – it was just beautiful. That made an impression.

You know, I still find youngsters that graduate from college who don't understand this. They can't tell you why the quadratic equation is solved by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. I have actually set that exercise to the 3rd years one time, to prove that if, and only if, this, and you would think that they could do that, but no.

How did we get onto that? You asked me about mathematics that I remembered.

PM: *The age at which you first noticed – but I think from your answer it is quite clear. It was starting with addition, although, if you are noticing that you were interested already at the point where you were doing addition – that is quite young indeed.*

AOF: I could see there was a trick to it, that it was interesting. I clearly had a talent for mathematics. I remember the nun setting us a board-full of problems and sitting down. I just did them all and folded my arms and she was surprised because everybody else was still slaving away for the rest of the hour but I had finished them in a minute or two. It was clear there was a difference in terms of how I could perform.

PM: *Did you also go to secondary school in Templemore?*

AOF: I started school in Templemore and then went up to Dublin to Drimnagh Castle – we lived in Walkinstown and I went over to Drimnagh Castle.

I met Brother McGrath the first time – my mother took me over to see if I could get into the school and he took me in and gave me an exam – some maths to do. He must have spoken to me as well to see if I spoke Irish.

You see I had gone to a school in Templemore that by a kind of fluke operated entirely in Irish – I thought that was the way the world was but actually it hadn't been going for very long and I don't suppose it continued but that was the situation then. As we walked in through the gate hand-in-hand, somebody said we have to speak Irish here and we did. For the five years that we were there we just spoke in Irish.

Anyway Brother McGrath put me into the A class – there was an A class and a B class – so that meant that I did the honours courses in various things. I did science subjects instead of doing commercial subjects and so on for the Leaving Cert. He was a big influence on my life. I had Brother Guilfoyle as well – taught me maths, Latin and Spanish – he was good – but Brother McGrath was exceptional. I published a book [4] in homage to him.

PM: *I saw that listed actually on your list of publications [5].*

AOF: He taught us religion and physics as well in much the same way. He had you buy a notebook and then he dictated. He had the diagrams on the board and wrote some formulae and things and we copied them into our notebooks; so I had three of these 120 page notebooks full of this material. He had it down to a fine art.

I believe he taught Latin as well but I didn't have him for Latin and I believe he was outstanding. He taught in the CBS in Ennis before coming up to Dublin and he had Flannan Markham down there, and Fr Markham ended up as a teacher in Maynooth – on the seminarist course in Maynooth. He told me once – Brother McGrath said: “Flannan Markham was the best all round student I ever had”. He said: “You were the best at mathematics but Flannan Markham was the best all-rounder”.

PM: *Which other science subjects did you do?*

AOF: We did Maths, Applied Maths, Physics and Chemistry for the Leaving Certificate and the manipulation was enjoyable. For example, proving trigonometric identities – we just were aces at proving trigonometric identities because he gave us hundreds of them to work through. It was an enjoyable kind of an exercise to develop that skill of handling these functions and the identities.

PM: *When you were thinking about going to university, had you already decided at second level that it would be mathematics you would study?*

AOF: It was kind of random in a way. I initially assumed that I would be an engine driver – something like that – and then I thought I would be a tradesman, like my father and grandfathers, so I was heading in that kind of direction.

At some stage a friend of ours, of my parents, said: “You know, they are recruiting people into this programme in Kevin Street which is really good for getting to be an electrician – you should send him down”. So I went down to Kevin Street one evening and there were dozens of people there and they were being advised by the staff in this big room. So when it came to my turn I met Tom Ambrose – yes, Tom Ambrose was there – and he was actually acquainted with my Uncle Billy in Roscrea. In any case, Tom Ambrose was there and he just asked me a few questions. I explained to him what I was doing and where I was – at that stage I was about 15 – and he said: “Go back and stay in school and finish!” He said: “Don’t dream of doing this, go back to school and stay there!”

I was interested always in understanding everything, so I wanted to understand the hard stuff. I was reading about things. I wasn’t spending much time on what was being done in school but I was using the library. I was reading about Biology, I was reading about Physics and Chemistry and so on and I was kind of surprised when they actually started doing some of the stuff in school that I had been reading about. I was kind of pleased, like the atomic theory and so on when that came up. It was interesting that they were actually going to do that in school because I hadn’t been depending on that.

Then I got to figure things out – what was hard. What was hard at that point were these new things in 20th century physics like quantum theory and relativity – which were clearly difficult things and I wanted to understand those.

I would say that was my most important motivation at that point when I was coming up to the Leaving Cert because I wanted to understand the hard stuff in physics, which is basically mathematical physics, I guess.

The Christian Brothers were good at getting people jobs – that is what they figured they were there for really – to train young people so they could get steady jobs and that mostly meant the civil service or teaching or whatever. So they sent you in to do the Junior X examination.

PM: *Which is a civil service exam?*

AOF: For entry to the civil service. So we did that – you wrote essays in Irish and English and you did something else – I don’t remember – and then you got allocated a place. You were 360 or ‘400 and something’ or whatever on the list and so you would get that offer in due course and that would be the start of a step where you come in at a junior level of the civil service and you would work your way up to secretary of a department or something – inevitable steps.

They also sent me down to do the Met Office exam – the Met Office had invented this concept of the Met Cadet. They had two positions every year and they had their own maths exam – it was just a maths exam – but they set it themselves. You did this and then they interviewed. They called the people who did well to interview and, if you were successful, you got one of these Met Cadetships.

What that did was that it paid you a salary, you were employed as a Met Cadet, you were paid the salary of a Met Assistant – someone who reads the instruments – that kind of salary – about £500 a year and your fees were paid to go to college and do a degree in physics related subjects. You could do Physics or you could do Mathematical Science, in any of the universities. So I was offered that.

The other possibilities were scholarships from the Council – we lived in the County rather than the City. The city had a number – I think they had about 48 scholarships – some number like this of scholarships – I think it was a multiple of 12 – but there were fewer council scholarships for college. Then there were entrance scholarships as well that a person could get. You paid for education, and you paid for secondary school. My parents had to pay £10 a year for me to go to secondary school, which was about a week's wages.

Most of the people who went to college were from a professional upper middle class kind of family. I think about 2% of the intake in UCD when I went in there came from the class of people who worked in any way at all with their hands, including tradesmen, and unskilled workers and so on. All you had to do was get 5 passes in the Leaving Cert, including Irish and Latin, and you got in. If you had matriculated, you could do whatever you wanted, that was the setup. There was one guy – a friend of mine – who like some people had trouble with Latin so he went to Caffrey's College for a year to pass the Latin in the matric – so he joined us when we were in second year – he arrived into first year. I think he took about six years to get through Pharmacy after that.

PM: *So during your college years you were a Met Cadet?*

AOF: I was working for the Met service from '64 to '68 – and I was a Met Cadet for the three years in college. That meant that I was on their time when I went to college. It also meant that if I won scholarships as I went through – which I did – I didn't get them. Actually I missed out on a substantial amount of prizes and scholarships as I went through. Then when I finished I had to work for them for ten years – that was the deal. We can come back to that.

PM: *Let's go back to your time in UCD. Did you go in to study Mathematical Science?*

AOF: That is right. With Maths and Maths Physics – they were the interesting things – you had to do four subjects so I did Physics and Chemistry. The Physics and Chemistry were just the same as the Leaving Cert – they were trivial – there was nothing new in those at all – so you just ignored those for the year and concentrated on the Maths and Maths Physics, which were hard and interesting. Then at the end of that year you could stop doing Physics and Chemistry and continue with Maths and Maths Physics.

Somewhere along the way – I think in about second year – I decided that the Maths was more interesting than the (Maths) Physics because it was being done better. There were very good people – David Judge was a wonderful teacher. I met an awful lot of mathematics first in the Maths Physics classes because they were all technique, so the techniques for differential equations and they would be using Hilbert spaces operators, things like this. But without any rigour, they wouldn't worry about it but they needed to use them so they used them. Things were proceeding more slowly in mathematics but they were proceeding carefully.

And after a while you get tired of this other way of looking at things – of doing things – and you wanted to do it right, and that became preferred as far as I was concerned.

Anyway, my job was to do the degree and then work for the Met service and I was quite prepared to do that. So I was interested in the (Maths) Physics all along and interested in the Maths. I had good people. Timoney's father – Richard M. Timoney's father, Dick Timoney he was called – we didn't call him Dick Timoney of course – he was teaching me, and Gormley and Ingram – Ingram was a Jesuit – and Franklin – this would be the Franklin before any Franklin you would know – it was the elder Franklin – he was David Franklin also but he taught Statistics – they were an interesting bunch of people.

PM: *What about contemporaries – did you have an interesting group of fellow Maths Science students?*

AOF: The class was small. Gormley did a thing called ‘clearing first honours’ at the beginning. So the first day of first honours you had about 50 people in the room – they were standing at the back and so on – and then for two weeks he just covered the board in some random hard story. In that case it was about identities – trigonometric identities – using De Moivre’s theorem and so on – and that whittled it down then to about 20. They carried on with the 20 and then he wouldn’t let about 10 of those continue into second year, so we ended up with a small class in second year. I learned recently that the Department of Education were pretty annoyed about the fact that he produced so few honours maths graduates [3]. The other universities graduated many more people with honours maths who became maths teachers, but Gormley was focused on matching the Cambridge tripos regime, and few of the survivors went into teaching.

Who was with me? Jimmy Fay was my best friend there – he lives in Canada now. He went into IT. Jerry Lynam also ended up in IT. Jerry went to America. There was John Bradley, who did a doctorate afterwards with John Miller in numerical and then he ended up working for the ESRI. He was a Professor with the ESRI. He used to do these economic forecasts, I guess they were – for the whole national economy anyway, programmed in Fortran.

Who else was there? There were clerics. Seamus Mooney was a bright fellow but he was a Holy Ghost to start with and then, when he stopped being a Holy Ghost, he went over to California. I met him again – he was doing a PhD in Economics in UCLA – so I was on his committee there for a while for a preliminary examination. He went to work for a financial company. He started working for WG Grace, I think they were called. He was actually in the World Trade tower when it was hit and walked out. He died a few years ago.

There was Seamus Hegarty, who was an Oblate novice out there in Stillorgan and he ended up as an education Professor over in England. He ran a large national centre for educational research in London.

There was Mike Norris, who went on to run the dot-ie internet domain. He helped me in the early nineties when I was administrating domains here for maths and computer science.

PM: *I know that you spent a year working in the Met service after your degree in UCD. Do you want to tell me how it progressed from there or what changes happened during that year?*

AOF: Well I was interested in the subject and they were exciting times in meteorology because people were starting to use computers. There was a method that we had – a manual method that we had – for forecasting, which was reasonably good – at that time the numerical methods were not superior but they were going to be superior – that was clear – and so this was an interesting kind of a development.

I got interested in computing because Ingram taught us how to program in Fortran when I was in second year. I had some facility with that. So I was following these developments and reading the literature. We had a subscription in the Met service to the Quarterly Journal of the Royal Meteorological Society and there were interesting developments taking place in terms of modelling the atmosphere and so on. I realised that actually the Met service at the time wasn’t terribly interested in this kind of thing. They wanted me to forecast but I looked around me and all the other guys were actually in this thing for the money – it was a job – they were interesting people but their satisfaction was after work. They finished work and they went off. One guy was writing a doctorate in Philosophy, another guy was into Drama – this kind of stuff going

on in their lives. I was 20 and my life – that wasn't for me – it had to be everything. It had to be what I was doing. . . I might do this or I might just join a religious order and become a monk or something. But what I was doing all day had to be the thing. This wasn't working for me so I resigned. I said: "I don't want to do this anymore, what do I have to do?"

So I went up and met Transport and Power's Establishments Branch and we agreed that I would pay them back all the wages they had paid me when I was a student – refund them all that – so there was a bill for this.

PM: *Sounds like it could have been substantial?*

AOF: It was. It was actually less than I had foregone in scholarships coming through on the way up, but never mind, that was the deal.

So I agreed with them that I would pay this over three years and I did that. That was the way I got out of the Met service and I went back to college.

Gormley who was Professor of Maths in UCD was sympathetic to this of course. If I had started straight into the Masters (the year before) I had the scholarship – £550 or something for the year – but they weren't going to give me that a year later so Gormley gave me some tutorial work to do, correcting work and so on, and Frank Anderson took me on as an advisor in the Computer Centre, so that I could get some income from that and I did the Masters.

Then at the end of the Masters I did the studentship as well – that is what you did – you took the studentship exam – and I went off to America and I started the doctorate. So a couple of years into that I paid off the Met service and we were flying.

PM: *I know that you went to Brown University. How did you end up going there?*

AOF: Again, I didn't understand much about the world. Maurice Kennedy was the Registrar and he was in the Maths Department – he had studied at Caltech – and he was teaching the Real Analysis Masters course, I think it was, which I didn't take. I took Complex Analysis and Algebra – you took two subjects for the Masters.

So Gormley taught Real Analysis off the programme – it wasn't officially part of the programme at all – on Saturday mornings because there was a man called McKenna – Joe McKenna I think it was. There was a fellow he liked anyway who had gone to work for Irish Life or something – he had gone to work in industry – and had to stop studying. Just for his sake, he put on this Real Analysis course on Saturday mornings; so three hours every Saturday morning. I went to that and Liam O'Callaghan and Joe and we read Hewitt and Stromberg.

Gormley also taught us German one evening a week for a while – he taught Liam O'Callaghan and myself German – bought us these books – I still have it, *Deutsche Sprachlehre für Ausländer*, there somewhere – enough German so we could get by.

Maurice Kennedy – I never had Maurice Kennedy as a teacher – called me in to his office and he said: "Go to America, because it is better – the graduate study system is better in America". He said: "Go there and they have this setup where you can get support, you will have courses and you will be properly prepared". So I said: "Fine, I will do that".

When I was doing the Masters I had two subjects. For Algebra I had mainly Tom Laffey – Tom Laffey is a fantastic teacher, as you know, but for Complex Analysis – I had already had a whole year of complex analysis from Gormley – we had this guy, Ernie Schlesinger, who was a student of Ahlfors and came on sabbatical for the year and so he taught the Complex course. This was really a second year of Complex Analysis on top of the other one so we learned a great deal of Complex Analysis from Ernie. I asked him – I said: "I like this stuff – where do I go? What is the best thing?". So he

advised me which schools I might apply to and so I made a list of five and applied to them.

One of the things I was concerned about was I had heard these stories about American graduate schools that there was this terrible competition that went on. People were viciously competitive, the students were competitive with one another and it was hard to get supervisors to take you on, and so on and so forth. I was concerned about that and so that dictated what happened next.

Harvard was very sniffy. They had this thing where you had to apply to apply and I didn't like that. That seemed to me to be pushing it a bit, so I had a few places – Michigan was one of them as well. Ernie had suggested places that would do things like Complex Analysis. Now he did say actually that things had moved on so mostly we talk about function spaces rather than functions – that is the centre of interest – so you might be thinking of looking at that – so he had suggested looking at these different places. Maryland was one of them and Brown and Michigan.

So it boiled down to it that Brown just did a better marketing job as far as I was concerned in that Schlesinger had a friend there – Robert Accola – who was another student of Ahlfors and he sort of wrote a “Dear Bob” letter to him.

I also got a letter back from the Foreign Student Officer, Mrs Burnight, saying that she had visited Ireland and she had had a very nice time. She was there the year before and the weather had been lovely. I wrote back and said I remember that week – that was great. She got the joke and invited me to drop by when I arrived. It was just personal and it seemed nice so I thought that sounded like a place that I could work in and enjoy and it was. It was terrific.

There were about 4500 under-graduates and 1500 graduate students in the place and there was a tremendous atmosphere really because there were these people, who were doing all these other subjects, that were interesting to talk to and they had a programme of Colloquia – people who would come to visit – they were all superstars that came to visit.

Off the top of my head, there was Lang, Grothendieck, Deligne, Douglas – Jessie Douglas – Hörmander, Segal, various top complex analysts would come by.

PM: *A very stimulating environment then?*

AOF: Yes. There were about 30 staff and they were world class people and they all came to coffee at 4 o'clock in the afternoon every day so you got to meet them. If you were stuck on something or you wanted to hear about something, you could go and you could ask them.

There was a separate division of Applied Mathematics. Wendell Fleming had a position in both of them. You know the way, once you have separate departments, they start fighting each other but there was a connection. I went over there to learn APL and to listen to Lorenz – the guy who talked about chaos. He had discovered the chaotic behaviour of the weather system – discovered chaos I guess – and I went over there to listen to him talking about the impossibility of forecasting over a long time.

On the Mathematics staff there were Katsumi Nomizu, Allan Clark, Jonathan Lubin, Paul Baum, Alan Landman, Gayn Winter, Robert Ferguson, Tom Banchoff, Michael Rosen, Bruno Harris, Yuji Ito, etc, lots of talent and variety.

Back along Hayman had been there – before my time – and Tamarkin – these kind of people. They had benefitted from what happened with what the Germans did, what Hitler did; so they had all these people. They were a very strong group in Functional Analysis and Banach algebras – that was the reason I was going there in the first place – that is why Schlesinger picked that particular school. So you had John Wermer and Andrew Browder and Brian Cole and Barnet Weinstock, who was in several complex variables, and Eva Kallin.

You see, at UCD I had been introduced to algebra by Fergus Gaines in the first place. He was the first guy who told me the definition of a group. I thought it was great. As I say I had a year of algebra then for the Masters with Fergus and Tom – that was all good stuff. It was skewed; there was a lot of stuff about finite groups and that but as far as rings went. . . You see the influence there was from non-commutative rings particularly, so Herstein and Kaplansky, etc. – Fergus had been a student of Olga Taussky-Todd – and they hadn't told me anything about commutative algebra and recent algebraic geometry, so that was a revelation.

The American system is great – you get to take these graduate courses. They required you for the prelims – they had to do prelims after the first year – they required you to do Algebraic Topology because that kind of stuff – homology and cohomology and so on – was really established as an important thing.

So in my first year then I didn't take the courses on Complex and Algebra that they were going to examine for the prelims because I had done that stuff but I did the Algebraic Topology with Bruno Harris and I did the Real Analysis course with Herbert Federer as well which was fascinating. Basically he was teaching it from his book on geometric measure theory [2].

I took a reading course with Barny Weinstock on several complex variables and so that was when I started learning about sheaves and the Cartan theory – the Oka-Cartan stuff but then there was a lot of that in the air around the place, that way of looking at things.

I listened to Landman and Fulton on Algebraic Geometry, Wermer on Potential Theory, Ito on Ergodic Theory, and Accola and A.O.L. Atkin on modular functions.

I took the PDE course twice, once from Federer and once from Walter Strauss. Walter Strauss was from out of Courant – he was that kind of background – a very standard Courant-Hilbert kind of approach to things.

Federer liked to always get down to the fundamentals so there is a point in the theory of hypoellipticity you need to actually use some stuff which is nothing like analysis, which is the Seidenberg-Tarski theory that goes behind that.

The basic idea is you want to know if you have some kind of a semi-algebraic set then the way it grows as you go out towards infinity is controlled by a power, and so, to prove that you have got to use the Euclidean Algorithm in the non-Euclidean setting of more polynomials over several variables. Federer went right through all that stuff. He took a couple of weeks just doing Seidenberg-Tarski and explaining all that stuff and it was almost like logic more than anything else. Quite different from Walter's course. I mean they would have overlapped on basic things like Cauchy-Kovalevskaya and so on. They diverged quite a bit then in terms of where they went with that.

The other thing I got from Federer was he introduced me to Hausdorff measures, which was a fantastic idea.

I was taking a reading course with Wermer on, well whatever he wanted really, but he gave me a paper by Gamelin and Garnett about Dirichlet algebras. I guess he had it to referee – I am not sure – it was a pre-print from Theodore Gamelin and John Garnett about Dirichlet algebras and they were using capacities to do that.

Simultaneously then I am learning about Hausdorff measures from Federer, so I figure, ok, here is what we will do. These capacities must have some kind of a dimension – a relationship to Hausdorff dimension – so we will try and figure out what that is. So I figured out that the analytic capacity should break at dimension one, and I cobbled together a proof. So I took this into Wermer and I said: "Look, I think this is what happens", and it turned out this was a known thing, which had been proven by Dolzhenko already but it was the start of my interest in capacities that came about at that point.

PM: *You ended up doing your thesis with Brian Cole?*

AOF: First of all I wanted to work with Federer because I liked him but he had three students in various stages – he was just too preoccupied – so it wasn't going to work. This was the beginning of second year.

I am looking at who else I would work with and Brian was giving a course and when I heard about it there was just one student that was going to be there, Richard Basener, and it was on Rudin's book on Function Theory in Polydiscs – they were working their way through that – so I joined that. I listened in and liked the way he operated; so I asked him if I could work with him and so he took me on. He was good. He was a good listener for a start, eccentric in terms of work time – he was nocturnal – so he stayed up all night working and he would come in about lunch time with a flask of coffee.

I met him once a week for the whole afternoon or whatever – we would talk for the afternoon – and he would stay there all night then after that – sleep in the morning – but he was very good. He taught me a lot – we had a lot of interesting conversations – and he listened to what I had to say. He did make some suggestions but I was really pursuing lines of interest to myself. I had a thought about a way things could be done. At some point I formed this idea that the capacities... you see Gamelin and Garnett had used *two* capacities. They had used the Ahlfors capacity and they had used the continuous analytic capacity as well; so I realised that this was flexible. There is a mantra that there is a capacity for every problem and so I figured, ok, let's systematise that.

I was influenced by reading Constance Reid's biography of Hilbert at the time. The way Hilbert operated – he sort of axiomatised and systematised things – so I had this idea that this is how you do things. Also it was in the era of Bourbaki so people did all these kinds of things like that.

I approached this then by trying to make it like that. So I would say we will have categories and we will have functors and there will be functors from a problem area which would be one kind of a category to a capacity which would be something else and the capacity would capture the thing.

So it was a cosmic scheme and basically it is a sound enough scheme and there is a lot of work to be done on it still but I was pursuing that circle of ideas. I could see that there were connections into complex analysis but you could use it also for PDEs – for elliptic PDEs as well – and you mixed it with function spaces.

So basically for an operator and for a function space you are going to have a capacity – from the combination of these two you are going to have an associated capacity and the capacity somehow captures most of what you need to know about that function space and operator. Then you would, if you were comparing two operators or comparing two spaces, looking at approximation problems or removable singularities problems or whatever, that the capacity would be the thing that you could use to combat that. So that was the cosmic scheme.

Brian suggested something to me – he suggested that I look at the Hausdorff measures associated with the Gleason distance. The Gleason distance is just the metric of the dual space of a uniform algebra. So you could look at that metric on the maximal ideal space and that made a metric space out of it and he asked "what could you say about that?" I decided that actually you couldn't say a whole lot about it but it did eventually lead me some years later to a result where I looked at the variation of the Hausdorff content as a function of the dimension. So I have a paper about that – I don't know if there is anybody else who has written about that – but it is a paper, in the JLMS I think.

You see when you actually go to apply these capacities to problems what turns out is that the Hausdorff measure is not the most suitable thing because the Hausdorff

measure is very often infinite on bounded sets. It is better to have something which is finite on bounded sets; so the content does that. It is the size infinity approximating Hausdorff measure in Federer's terms and that stays finite on bounded sets.

If you look at that, since it is finite for every dimension, you have a function of the dimension there and you could ask about the variation of that function of dimension and its continuity properties from right and from left and so on; so I investigated that.

I was able to use some lemmas and things that I had worked out to approach Brian's problem to get a result there, because what you are doing is all about coverings – the contents are defined in terms of coverings – so if you are looking at the variation – varying the dimension – then you have a family of coverings. You want to extract from the family of coverings some kind of a covering at the end which will do something so you are looking at that kind of a problem where you are extracting a convergent sequence in some way from a family of coverings. What you don't want is that they all end up being points or something at the end. That was the problem that we hit when I tried to do the thing with the Gleason distance – they tended to end up being points – but, if you just did the straightforward question it was ok.

PM: *After your PhD you spent some time in the States before you came back to Ireland?*

AOF: Well I was interested in these analytic capacities and capacities generally and expertise in those – where was it? There were people in Russia, there were people in Sweden and there were people in Los Angeles – that was the universe as far as that went. You had Vitushkin and Melnikov – these kind of people in Moscow – you had Carleson and people around him, Hedberg, in Uppsala and you had the school in Los Angeles. Oh and there were people in Indiana as well – Thomas Bagby in Indiana had worked on the capacities for L^p spaces – L^p analytic spaces – so I was interested in those.

I tried to go to those places. I applied for positions in – I didn't apply to Moscow – but I applied to Uppsala (Mittag Leffler) and to Indiana and Los Angeles. I was particularly keen to talk to the guys in Los Angeles, Gamelin and Garnett, because I had put a lot of energy into studying their work. So it worked out that I got the job in Los Angeles. I had a visa which said I had to go back home after finishing so I had to explain to the embassy that I needed to go for a couple of years to Los Angeles instead.

I met a friend afterwards who said: "How did you do that?" I told him and he tried to do the same thing, but he was in English Lit, and he wanted to go to Bates College and they said: "No, you can't do that". I am not sure what swung it but I suspect it might have been when I said that the expertise on this is in Los Angeles and Moscow.

PM: *The fear of Moscow maybe!*

AOF: It might have been crucial – I don't know – but there was no problem. They just listened to me and they said: "Fine", so I got the permission to get the visa for an extra couple of years and I went there. I was never intending to stay in any case. I was planning to come back home but I figured I needed to talk to people because the guys in Brown didn't really do these capacities. They were very, very strong on the functional analysis and the abstract end of functional analysis and Banach algebras and so on but they were not into the hard analysis of capacities which is kind of a black art – a lot of people shied away from it because of that – but these guys, Gamelin and Garnett, they were really stuck into that so it was great to be able to talk to those people.

I was always planning to come back and so I was building up. I was photocopying things that I couldn't read then but I figured I would read later when I got back. When I was at UCLA I was copying away anything like that and planning to ship it all home because I was coming back here where there would be nothing. None of this stuff would

be readily available – I would be sending for inter-library loans for weeks. That was the scheme. It was good, they had an excellent programme there of visitors. It was a bigger school with I think about 70 or 100 staff, a very good library. They had a very good library because something to do with... I think it might have had to do with the atomic bomb programme or something.

You see UCLA was one of the first places in on the ground floor of numerical computing and they had kind of lost that when I got there. They were trying to re-establish it – they were trying to hire Garabedian for example because of that and he was playing them around because he wanted to improve his position somewhere else – but they had a super library – just a maths department library – very well equipped.

Richard Arens was there. He would have been the grand old man of that material. Sario was there of course as well, but Sario didn't come in very much. Then there were people like Redheffer and these guys – Coddington and Redheffer in differential equations. They had visitors – Takahashi – these C^* -algebra people came around to visit – as well – Costant – so I learned a bit about that end of stuff as well.

The Banach algebra meetings that went on every two years for ever – they started that. That is where I met Garth Dales in my second year and the first of those meetings was at UCLA. Sandy Grabiner and Joe Stampfli came and Bill Badé came down from Berkeley. I shared an office with his student, Fred Dashiell at UCLA.

PM: *The whole transition then from the US back to Ireland and your position in Maynooth – there has to be an interesting story there?*

AOF: Well, as I say, I was always intending to come back here. Now I did make applications around the place just to see what would happen. You will observe I am moving west, right, so I had gone to the east coast and west coast – I think it is a Celtic tendency to travel west – so I did actually get a job offer from Hawaii at one stage as well and that was kind of attractive. I think it was that going westward. There was a guy there called H.S. Bear – I guess he did Gleason parts – so that was tempting. I had this interest in warm places, sunny beaches and things like that; so Hawaii was attractive from that point of view.

I had offers from several places in the States as well but I would have had to get some special kind of visa to do that but I suppose it would have been possible. There were these H visas or something which would do it if they made a case for you.

Basically I turned them all down because they are out of synch with Ireland, so you get the offers in America and you have to respond before things are settled in Ireland so I just decided to burn my boats. I applied for a Department of Education – the Department of Education had one post-doctoral fellowship in mathematics which was designed exactly for people like me. You could come back to Ireland and you could hang around until a job turned up because the jobs were few and far between and so I was pretty sure I would get that.

Then Trevor West wrote. Trevor West would land in – he knew people everywhere – and say “I am staying with you today”, or whatever, that kind of a way. So Trevor West arrived in, introduced himself and stayed with us and he took me under his wing. He wrote to me and said: “Apply for the job in Maynooth”.

Oh I applied for a job in Cork as well. They called me to interview. That was when I was at Brown – when I was finishing up at Brown I applied for a job in Cork – Gormley told me to apply there. They called me to interview but I decided not to go – I decided I would go to the other place.

Timoney – Timoney's father wrote to me as well – Gormley was dead at that stage – and he wrote and said “You should apply for the job in Maynooth, because you won't get it but you will be in a strong position if you are shortlisted for the next statutory lectureship that comes up in UCD.” Trevor wrote to me and said: “Apply for the job

in Maynooth and put me down as a reference” so I did that and then I was called to interview – what was I – 27?

As far as I was concerned I had won because I had been given a free trip home – that was progress – that was all I could expect to get out of it – so I wasn't at all worried about it. There was no pressure. So I came home – I was home for a week or so – and I took the bus 66 out to Maynooth and went for the interview. Trevor marked my cards as well. Trevor had a football team and I had to go and play football with his football team up in Alexandra in Ballinteer. He said: “You are a professional – just talk about your stuff”.

Of course I had very fixed – formed – opinions about what should be done, like what I would do differently if I were doing my own education over again, what people should know about and what was important in mathematics and so on. I had views about this – perhaps premature – but anyway they were views.

So I went down and enjoyed myself – it was fine. There was a board with A.J. McConnell. It was the first time they had open competitions. It used to be that the bishops would get together and decide who would be appointed. Way back in the 19th century they used to have concursus – candidates would come – the whole college community would come and listen – they would have this debate, where people would throw questions but more recently the bishops just decided what happened.

Maynooth had just opened up to the world at large – that was a good thing. When I was away in America, I thought Galway would do, because the wind comes in from the ocean, and the air is better in Galway; but Maynooth is upwind of Dublin as well so I figured that was ok; so it was an opportunity.

They had this idea – they had a board with a non-voting chairman.

PM: *That was A.J. McConnell?*

AOF: That was A.J. McConnell, and he was Provost at Trinity at that time and he was impressed that I had burned my boats. At this stage I had refused all offers to go elsewhere. I had this thing in my pocket now – the scholarship.

PM: *You had been offered that?*

AOF: I had been offered that and my mother was scandalised. She said: “After all you have done that is all they are going to pay you?”, but nevertheless I was coming back with that regardless. That probably had some influence. They were going to get me – I was definitely coming. Tom Fee was the President and he was mostly interested in whether I spoke Irish or not, and was Irish – I think that was important to him as well. So we had a little chat in the first official... and that was fine. I'd kind of kept it up. I bought a record in Donegal Irish on my way out in Shannon and played it obsessively when I was over there. I didn't know any Donegal Irish when I went out to America but I had learned it by the time I came back.

The rest of them then – they are all dead – there was John Lewis, Gerry McGreevy and Joe Spelman so Director of the School of Theoretical Physics at DIAS, Professor of Maths Physics, Professor of Physics and David Simms.

So we just had this conversation. The next day I gave a seminar over in UCD just to tell them about stuff and David whispered to me that I had been recommended for the job. I wrote a little article about this.

PM: *I didn't spot that actually, as I would have been interested.*

AOF: It is called “An Chéad Ollamh Tuata” – you will find it on my website [5].

I just told this story about the interview process because it showed how they were just feeling their way in Maynooth at that stage.

PM: *When you came to Maynooth, it would have been a small department?*

AOF: There were just two other permanent staff – David Walsh and Richard Watson – who were lecturers in the department. They had a temporary – a one-year – position the year before. The previous Professor was J.J. McMahon and he left the priesthood and resigned his position. He was gone off to Nigeria. He spent a couple of years in Nigeria and then he was hired in Limerick after that. He died not long afterwards. I met him at the Institute symposium maybe once or twice but then he got cancer and died. He was, apparently, quite an eccentric character. There are some amusing obits of him in the Bulletin.

The two other guys were both hired in the early '70s – I think, David about '72. David was a student of Finbarr Holland. They hired a few people in '72. Richard was actually in UCD in a temporary position the year I did the Masters. He had gone through Maynooth seminary and did the Divinity degree as well. Then he went to England to study after that. He studied in Warwick and Swansea and came back and he was appointed in Maynooth; so I had those two guys.

Initially it was a bit challenging, in that I looked at the programme and decided it needed beefing up. The system had been to run things in such a way that for most departments just a man and a boy would have been fine, because you had a first year general course and then you had an extra hour for the honours – and that was the first year honours course – and then you had a cyclic second and third year general course and some extra hours for the honours course on top of that. You might have five lectures a week to give, you see, for that and four for the other; so it is a nine hour teaching load. So with two people it is quite comfortable with that – they can play golf in the afternoon, that kind of thing.

Whereas I looked at our programme. We had first of all an entirely separate pass and honours, so straight away we have got a heavier load than they have – number one. Number two, there wasn't enough being done – I didn't like the cyclic thing. I said we have got to get rid of the cyclic thing. They have got to do second year and then third year; so first of all we have got more to do in second and third year because we have four plus six – I am raising it from five to six – and then we have to double that, so that is 20 hours plus the other. The other departments – the experimental departments – had no honours degree at that stage. The only honours degrees that we had in scientific areas were the mathematics ones with mathematical physics, but they were keen to do that. As a first step to that, they wanted to have a fourth year on top of their general degree and so they introduced this course where there would be a fourth year.

We had to provide a fourth year with that and, of course, we couldn't use the one we already had because it was too hard for them. Even in fourth year they weren't going to be ready for our third year course so we had to do a special course for them. That was another four hours on top, so it soon totted up to 41 hours – we always ran a taught Masters course as well.

Now, unwinding the cyclic thing – that took a couple of years – and we did get a staff member. They had a kind of a tradition that, if you got a new Professor, he got a new staff member and then they thought he was happy after that; so we hired Dave Redmond.

So we had four staff to teach the 41 hours so that was roughly ten or eleven hours each – that was the deal. By modern standards, this is a lot but that is the way we worked. It got the thing going so that it was credible.

PM: *At that stage, it was probably very comparable with the other universities though?*

AOF: I think Gormley and Timoney and those guys were working about 14 hours anyway, until the three young staff came when I was, I think, in second year or third year. Fergus Gaines and Tom Laffey and David Tipple came together and that was

something that Jeremiah Hogan did – he was the President. He had this idea of what you call College Lecturers – it was an invention.

What they had up to that point were statutory positions – there were Professors and statutory lecturers – and perhaps temporary people – but he invented the ‘college lecturer’ and he allocated three of them to mathematics. Before that they just had Gormley, Timoney, Maurice Kennedy, Stephen O'Brien and Franklin – that is the whole lot. They had to do the science degree, the arts degree, the engineers – the engineers did four years of mathematics back in those days – I am not sure what else they had to do that was separate. Maybe architecture and there was something else that they were at.

PM: *Probably commerce or business students as well?*

AOF: They had BComms – that is correct. They were flat out. I mean they were crippled – buried under a load of work – at a killing kind of a pace. It is a pity really in retrospect. I mean Gormley had potential, he had written about half a dozen papers. He obviously had a lot of talent and he had a lot of stuff that he could have done. He wrote to me in America – he was thinking about the stuff that Loomis had done. When we read Hewitt and Stromberg he was pursuing the thoughts that he picked up there and pushing out on that but I don't believe that was ever published. His published papers are all from earlier on. He has a paper on quaternion linear fractional transformations with applications to special relativity. His doctoral thesis was on differential geometry.

PM: *Over the next decades then, there were obviously lots of changes in all of the universities but, in particular, in Maynooth. Do you want to summarise the trajectory there, from arriving into a department with just two other staff to the makings of a modern department of mathematics?*

AOF: Well, I had to be rude to everybody until we got enough staff to work six hours. I figured that was the target as far as I was concerned. If people could have six hours then I expected them to have time to do research as well as that. I didn't think it was reasonable that they should have more than that and they didn't generally understand that so I had to be rude to everybody for a good while and I think it was the 90's before we got to that level.

PM: *That is a good number of years of being rude to people!*

AOF: Yes. I can remember these various incidents – one time I referred to the academic staffing committee – of course it used to be more democratic – the academic council would actually decide this stuff – nowadays administration decides all this kind of stuff – but the council would really determine the policy.

The Council had a committee of course – an academic staffing committee – that they would refer this to and they would draw up a proposal for what we would do in the way of hiring. I described them one time as a “cabal”; which is a term technically speaking where you are referring to a black mass kind of group, people who were doing something devilish. Matt O'Donnell was very annoyed about that and insisted that I withdraw “cabal”. Anyway you had to go at them. Eventually it got to a level where I thought it was reasonable.

As I say the youngsters nowadays are used to a different world and would find this excessive perhaps but, when I was in America, the standards were that, there were teaching institutions – there were two year colleges and four year colleges – and there were universities – and in these different places expectations were different. Twelve hours was regarded as a teaching load for someone who was just teaching and, for someone who was expected to get some research done, six hours was regarded as the

standard so I took that as a basis. I certainly found that I could work like that myself and it was ok.

I suppose people put up with my bluntness partly because I was energetic and helpful: I looked after the purchase and installation of the first computer in College, set up and ran the Computer Centre for a while, took over and organised the university timetable, ran Computer Science for a while, volunteered a lot, etc.

PM: *Do you want to say anything, Tony, about your private life and everything going on there at the same time? At some point you obviously got married and had a family. At what stage did that happen?*

AOF: As I say, I sort of thought I might become a monk, early on, and that lasted until I met Lise. It was always in the back of my mind as a possibility, until I decided that I wanted to marry her. I remember that was a bit of a crisis for me in the sense that it was a shock to find that actually that was going to happen.

PM: *That, by definition, is a very romantic whole change of heart to some extent! I mean, being a monk and being married. . .*

AOF: I thought women were wonderful and interesting to talk to and so on but I wasn't really definite about what I was going to do about that until I met and fell in love with Lise. I remember in the first month or so – the first few weeks after that – being somewhat disoriented, because I was re-orienting my direction and getting used to it. So she was a huge influence in lots of way. She introduced me to choral singing, for example. Probably I had a much bigger social circle, a richer social life than I would have had otherwise. I learned a lot of things from mixing with Lise – Lise brought me into a whole world of possibilities. She is perpendicular to mathematics [6].

PM: *Did you meet Lise in the United States?*

AOF: Yes. When I went over in September – you see I did the Masters exam, the studentship exam, one week and went to America the next week and the results didn't come in for another month or whatever. So when I got over there, I was supposed to go up to the Foreign Student Office to introduce myself so I did, and there she was. She was the President of the foreign students association – the International Association.

The foreign student officer asked her to come in that week and meet the incoming foreign students so, when I arrived up there, there she was sitting out in front of the office. They had given her a desk out in the concourse in front of the foreign student office. She was wearing this very fetching dress – she had bought these dresses in Africa I think or somewhere – they were very nice. They were short and they had patterns on them and things – African kind of patterns.

We hit it off because – well, she will tell you the story – but, she told me what she was President of and I asked her where she came from and she said she was from Seychelles and I knew about Seychelles. She was used to all these people who would say: “Where is that?” I can't tell you the number of times I have heard Lise say – she had a little spiel – “they are a group of islands about 1000 miles off the east coast of Africa”, and they would never know. But Ireland has this missionary diaspora so we have missionary orders who go all over the place and they have magazines – I still have subscriptions to the Africa Magazine and these kinds of things – so these came into the house. So sooner or later somebody writes two pages about Seychelles.

There was a Franciscan Capuchin magazine that came in and so I had read this – because I read everything that was available – and so I knew the basics about Seychelles when I met her and I was pleased because I was interested in hot foreign places as well – Paradise and all that sort of thing. Gordon, the guy (British General) who was killed in Khartoum (1885), thought that the Vallee de Mai in Praslin was the Garden of Eden,

he had this theory that that was it. If you know about Seychelles, it is famous, Vallee de Mai. So, anyway, that was the start of a beautiful business. We got – and we weren't quick about this – engaged officially in '71 and married in '72. She graduated in '71. In '72 I was still a graduate student – I graduated in '73. Now I had written the thesis. Basically I wrote the thesis – I figured out the stuff that was in my thesis — in my second year – it is a four year programme – the expectation was that you are going to be there for four years. The way I am thinking is I am going back to Ireland and it is going to be a wilderness; so I was in no hurry to go back. I needed to work on my inner fat and have that with me when I went back.



FIGURE 2. Lise O'Farrell

I had the results – I wrote them up in the first half or so of the third year – but I continued taking courses, learning more mathematics and using the library and I started writing papers outside my thesis as well; so that was what I was up to.

Actually when I applied for that job in Cork – it was in my third year when I think about it – Gormley wrote and said: “Apply for this job in Cork”, and he said: “Put it on a page and be brief” – that was the advice, right, and so I did that – a one page application.

Tadhg Ó Ciardha invited me to interview and I made arrangements that I could have gone then as well. I transferred the credit from my year in UCD to Brown so that I could skip a year if I wanted to but I decided, no, I am going to stay and just use the time and use the facilities here.

You see they had this library – the place is there since 1764 – and they had every book. Henry Pohlmann looked after the library and it was brilliant, they had every reference to everything you wanted – it was always down there in the library – so I was game for that.

Now Lise, she came home with me in '71 to visit and we made arrangements to get married in '72. Then we came back here and got married in summer '72 and then we went back to Providence. She was working in the public schools in Rhode Island for that year – she qualified as a teacher for the state of Rhode Island – she had the appropriate licence or whatever – so she was working there. She was actually supporting us – I mean she was the main bread-winner – I had a scholarship from the department which was fine but she was making more than I was. Then at the end of that year I graduated and we moved to Los Angeles and we went to Seychelles in the summer of '73 to visit her parents and family. On the way back in Ireland we discovered she was pregnant. We had planned to drive across the States but we had to skip that and fly straight.

We spent two years in Los Angeles and then we came here. We have been basically here except for some leaves ever since – it is 50 years next year.

We spent a term I suppose really in Connecticut one time from August – we went in the summer and stayed until Christmas. I had a year's sabbatical one time in '85/'86 and we travelled around. We went over to England for a while, to Cambridge, then IHES and then Israel and back to Cambridge. Other than that, we have been here the whole time.

PM: *You have seen a lot of changes, obviously in Ireland, but in the universities and, certainly, within mathematics within the universities over those 50 years. Too many changes to summarise?*

AOF: What can I say? The big change, of course, is the internet which has revolutionised the practicalities of doing mathematics, because it has made all this stuff available – it used to take two or three weeks to get something on inter-library loan – it mostly had to come from Boston Spa. You always had to work on several lines of enquiry in parallel because it could block on something where you just needed to get hold of something before you could continue and then you just have to wait for it to come; whereas now you can get instant access.

Communication – I mean I used to say that what we needed were research grants for phones. Bill Ziemer had a research grant for phone calls in Indiana when I went to see him at the time they offered me the job. That was very sensible because long distance telephone calls were expensive and that was the main thing – you could just communicate with people and ease that.

They never got that really – that it was worth giving people grants for. Communication is now – since email really got going properly – fantastic. The difference that has made! So in terms of the practicalities of doing work, that is the big change.

Administration – I don't think we should even go there – the corporate business that has taken over is just sickening to look at really. Among other things, I was the Jimmy Hoffa of Maynooth for a while. There was some iteration or other of the national wage agreements which was introducing something which looked as if it was going to be a disaster. So Vincent Comerford drew this to my attention and I agreed that I would have a go, so I became the person in charge of the IFUT branch here in Maynooth – for a while I was doing that – so I was dealing with these guys in the personnel office and the President and so on. It is just soul destroying looking at the way they operate but I think it will pass. I have hope.

PM: *Really, you do think it will pass?*

AOF: I mean the academic enterprise – this whole idea of the way we do things has worked for thousands of years – basically it is the same scheme – this managerialism and what not is currently triumphant but we will see the back of these guys eventually. They can't stop us doing what we love, that is really what it boils down to. We would actually do this for nothing – literally at the moment I don't have to – they will pay me the pension regardless anyway – we would do it for nothing.

Jerry Brown was the Governor of California when I was there – he was a Democrat I think – and they gave a pay rise at one time to all the public servants in California except the university Professors. He said: “They don't need it – those guys have *psychic wages*”.

People were outraged but he was right – it is the best job in the world. In fact, it is not a job. When I was young you had this idea that people did that – they got jobs – but what is a job? I had a job in the summer of my 5th year delivering frilly garments around Dublin on a bicycle and I realised that it was just slavery. I realised that because one day there was nothing to do – I was waiting for something to do – and I pulled out a book and started reading the book – and my boss said: “What are you doing?” And I said: “Oh, I am reading the book while I am waiting,” and he said: “You can't read”. So I would put the book down and do nothing, right, until he had something for me to do.

PM: *Even though there was nothing for you to do, they owned the time.*

AOF: They owned the time, so a slave. I decided, ok, this is not what I want – it is not the way. This life – this university academic life – is an example of a kind of life where you do what you want, you pursue your particular passion. The religious life is the same, you just do the one thing and focus on that and do it well.

There always have been people who did it – it was not fair – those people were rich. Nowadays I think we detect talent better than we used to do – so we do have a better chance of picking up if a young person has talent than we did before. There was a lot that went to waste and we have removed barriers. I don't know what you feel about this but I think it is probably better for girls – for women – growing up. I think they have better chance if they have potential to have that recognised and used and channelled. In general that is the case. We have removed obstacles to talent.

You see a lot of people around the town – a lot of older people than me – they had interests that they were not able to pursue. Like the fellow who used to fix my car, I remember when he was coming up to retirement he said: “I would love to do some third level”. Never got the chance and he was thinking of doing something when he retired and he would pursue that kind of an interest. You meet a lot of these people – you used to meet a lot of these people – even the old shopkeeper down in Rosslare who had just done the old Inter-Cert. There was an old Inter-Cert where they did things like Latin roots – that was a subject – but he thought about philosophy and had an interest. It is good that nowadays you can pursue these interests – you are more likely to be able to pursue them.

Our species is a kind of fluke. I noticed there you had a question about AI down at the bottom.

PM: *I did, yes.*

AOF: They talk about this AI catastrophe which is when the machines – the intelligent machines – take over us. I think the AI catastrophe already happened – it happened to God because he made us and we rejected him – we took over.

We are such a fluke – we are on this little skin on the surface of the globe and we are the only – certainly the only one left – thing like us, in that we have this capacity for reasoning – we have this intelligence – and what did they do with it? Like in the 19th

century, you take these people who essentially, among other things, had the capabilities of a general purpose Turing machine and you get them to work on one specialised task – they could be programmed of course for any task – but you get them to run a cotton mill or something or a loom – something like this – and you pay them to do that for 12 hours a day six days a week and they are wage slaves.

Now we can certainly produce all we need on earth with about 20% of our time and our labour so we should have an enormous amount of free time and labour to do other things after we have fed ourselves and clothed ourselves and looked after ourselves. That is what humanity is for – is good for.

PM: *Have you been involved in the second level maths curriculum in Ireland?*

AOF: Yes of course. I regard it as a duty of being Head of Mathematics here to take an interest in that, and so yes, at various times. There is a book [3] by Susan Mac Donald, just published by Logic Press, which tells the story of the geometry disaster – required reading I think for how not to run an Education Department.

So I was involved in various ways. One is that I was active in support of Paddy Barry and other people in relation to this geometry curriculum and the whole disaster around that. I was on the NCCA syllabus committee that brought in the Project Maths syllabus more recently.

Way back in the ‘70s Sean Ashe and I got involved – he was very interested in the fact that the lower Leaving Certificate course that they had then had a very high failure rate – both of them had a high failure rate – and he felt that the weak students were not been catered for and something needed to be done about that and so I was involved in that as well. I would have talked to the IMTA a couple of times and also just generally trying to contribute.

I have a paper about geometry there – school geometry – in the IMTA newsletter. I have a paper with Paddy Barry about the geometry as well and I wrote the geometry document for the syllabus. I had help but I was the principal author of that geometry document that forms part of the syllabus – “Geometry for Post-Primary School Mathematics”. Ian Short did the diagrams for me and Stefan Bechtluft-Sachs gave me a hand with it as well – but I put that together. That is what I call a Level 2 account of the geometry. That is to say you have a fully rigorous level, which I call Level 1. For this geometry programme – that is Paddy Barry’s geometry book [1]. This is a Level 2 account which is supposed to be a somewhat simplified version of that and suitable as a foundation. Then they needed text books which would be Level 3 – largely absent still. There is no Level 2 account of the rest of the programme in schools either; so there is a lot of work that needs to be done.

PM: *There is a review going on at the moment.*

AOF: The IMS Education committee are looking at text books at the present time – Ann O’Shea is chairing that. We have no system of validation or approval of text books – other places do. If you read Feynman’s book, for example – he was involved in the California State one for physics – I think for mathematics as well, back along. They had this setup where somebody says there are no mistakes in this book and it is suitable to use – that is the minimum – and we don’t have that.

The whole thing needs a lot of work. Or they take something off the shelf that is working somewhere else and is properly done, and translate it if necessary into English and use it. We ended up with this setup where it made no sense and it turned a whole generation of people off geometry – everybody hates geometry – just a disaster.

PM: *I know you retired in 2012 so that is before Project maths was fully rolled out. Do you have any views on how Project Maths has functioned since then?*

AOF: I don't know very much about what has been happening but I know a little bit and I am sort of worried about whether we will make it back to some sort of stable situation or not. I think huge damage was done because, as I say, a whole generation including all the presently active teachers, were cheated really of an education in geometry. It is very hard to work back from that situation.

I am worried about implementation, mistakes in the text books – mess everywhere – but we can but hope.

PM: *You played a founding role in the Irish Mathematical Society?*

AOF: Well I was around when it started alright – that is true – and I think the initiative was – was it Trevor, Finbarr, Tom Laffey and those guys who got it going originally – but I was around the place alright. I was Secretary and President at different times.

I think I had a hand in getting the September meeting off the ground – the idea of a scientific meeting. The Institute used to have these biennial meetings and everybody came. Like the BMC – everybody in Britain used to go to the BMC – at one time you would have about 600 people going. Now it has become the kind of thing where people come who are interested in the particular topics they are going to talk about. I suppose that is a general phenomenon – people don't like these broad meetings – a pity.

The Institute's symposia – everybody who was active in mathematics and mathematical physics in the country used to come and then they stopped doing that, when John Lewis finished, I guess. There was a gap there and I thought the society should do that.

So we had this idea of the scientific meeting – the annual scientific meeting – so they have been useful and good I think. Some very good folk came. Fred Almgren came from Princeton. I think Wermer came to one of those as well.

PM: *I have something here to remind me to ask you about the Hamilton Walk, which seems to have developed its own life now.*

AOF: Well that is a thing where you have something concrete and tactile that you can associate with mathematics. That was a thing. Hadamard has this book 'The Psychology of Invention in the Mathematical Field' where he talks about the way you work. I mean how do you do mathematics well – you take something and you think about it and you think about it hard for a while. Then maybe you don't – maybe you just let it sort of stew for a while – and then at some point or other your subconscious works away once you have got it started and then sooner or later the thing pops up into the forefront with some kind of progress.

There are a number of events that are recorded – we have reports of this – and one of them is the Archimedes in the bath business. Then there is the story of Poincaré stepping off a bus and inventing Fuchsian groups or something. And then we have this Hamilton story and the bridge – this moment when he suddenly realised that this is how it should be done.

The bath has gone and the bus has gone but we have the bridge still so we have a thing – a spot – and it does draw people. We have arrived at the bridge and found other people there, independently arriving from other places, so it is a thing. I thought that was good.

I did take an honours class – I had a small honours class years and years ago. I had a headache and I put them in the car and took them over there one time and then the thing started regularly. I believe it was 1990 that it started being an annual thing.

I went over to Dunsink and I talked to Ian Elliott – God rest him – who was the astronomer in charge over there – he wasn't the Director – and he was very helpful. He pointed out and explained to me about the geography of things. We had to pick a route. It was either down Dunsink lane or across the field, so we went for across the

field – off road – actually there were a number of obstacles down Dunsink lane in those days – and mapped all that out.

So I wrote something and sent it around the Maths Departments to tell them about it and the thing kicked off. David Simms and Nigel Buttimore turned up on the first day as well and we had a walk – the rest is history. It is nice.

It gave a focus. Maths Week then – they decided to build it on the 16th – to have the week embracing the 16th – and DEPFPA Bank came along looking to support mathematics and so it was natural to fold that into Hamilton day as well and have it be a day for mathematics.

PM: *I did have that question, which you have partially answered, about recent developments in AI. We are seeing in the universities now massive effects on how we can do assessments. Do you have any views on that?*

AOF: Well, I am not in any way an expert in AI. I have had a look at these generative things that are available and they are very limited in what they can actually do; so we are a very long way from real AI in any kind of Turing sense – nothing like that. I wrote some little note about it in the last editorial I wrote for the Bulletin, where I just described some experiment there and I have had another look or two since. I think it is well well short of actual artificial intelligence and it is over-rated in terms of expectations.

The thing you mention is right that the students can copy things – it was already beginning to be a problem anyway that they were copying things and it has undermined the whole idea of having continuous assessment in tutorials. We didn't have any of that when I was young. We didn't have any CA. In fact, there were very few examinations and we gave something up when we started having all this CA – we gave up the business where a student was just left alone.

When I went to college I spent a great deal of my time learning about music and philosophy, ancient classics – all this sort of stuff – and that didn't matter in terms of assessment. I could just do the examinations at the end. The final examination was at the end of the summer for both the Bachelors and Masters degrees; so you had a great deal of time to pursue other interests and then do this exam. Whereas (now) they are being monitored and regimented a lot – forced to do things as they go along and that is the downside.

I introduced all the tutorials and things when I came here because I thought it helped the students; so we set up this whole elaborate system now which is there for tutorials and set homeworks and all the rest of it. It was well-meaning but whether it is really better or not I don't know.

I mean I am not convinced about education. I think you can really divide the populace into the people that can't be taught and the people that don't need to be taught, they teach themselves; so the most we can do is point people in the right direction. Standing over them – I don't know – and this CA system has always been open to abuse – it has always been possible to copy and it has become worse and worse.

PM: *How would you describe your research style?*

AOF: Well what I do really is move out from things that I understand to the things that I don't and there is a nice principle which is that you are never very far away from the unknown stuff – the frontier is very close to anything at all. So I think the way I have gone about it is to try to understand things – to try to get right down and understand them and as soon as you study anything in sufficient detail, you will become aware of problems – things that you don't know that are around about that. And then I just operate by writing down any thoughts that I have and keep them. I have stacks

of paper in there and I have notebooks and things and I just move around these things then and tackle them as I can.

There is sort of a chain of connected things that I would have worked my way around [5], like capacities connected with length and area, rectifiability, dimension, algebraic structures, algebras, modules, groups, Hausdorff measures, kernels, Suslin sets, Polish spaces, extension problems, polynomial and rational hulls, function spaces, derivations, categories, exceptional sets, removable singularities – all kinds of problems about singularities – and then from singularities for holomorphic things to elliptic things, approximation problems. I always liked approximation problems.

It is a matter of taste what you do and what you like. And I always liked this business of approximation. The first approximation theorem I learned was the Bernstein approximation theorem – Gormley taught us that. We had first a raw observation that if you take a Taylor series that it usually doesn't represent the function. It might but it usually doesn't so then what do you do? The Bernstein thing was the first example of a theorem that says, well ok – it is actually Weierstrass's theorem – that you can approximate any continuous function with polynomials on an interval. Weierstrass's own proof of that is one way – Bernstein's proof is a different way – a limited way in some sense because it has this positivity thing that it goes with – but I liked that kind of thing.

So a theorem that says that we can approximate all these by rational functions in these circumstances or we can approximate by functions holomorphic in a larger set or whatever it is. We can take away the singularities and still approximate – I like all those kind of questions. I would regard those as sufficiently interesting by themselves.

Garth Dales asked me – he and A.M. Davie had these spaces of infinitely differentiable functions on an interval, where they made Banach algebras out of them by having some kind of a growth condition and a suitable norm condition that gave some multiplicative norm, and he wanted to know if the polynomials were dense in those algebras. That's enough for me – if you asked me are the polynomials dense in that – I like that question – and so I thought about that until we solved it. It took a long time before we saw what to do but eventually the penny dropped and we saw a way to do it.

Then you have got this business of special sets – Cantor sets and sets of convergence. Then potential theory, singular integrals, measurability and the whole business then of functional analysis, algebras of functions.

Browder taught me a functional analysis course in my first year at Brown and he did the Gelfand theory of commutative Banach algebras and that is just so beautiful. What he is doing is he is taking the stuff the algebraic geometers have done and he is transferring it then to this context of uniform algebras but you get things like Wiener's theorem – that a convergent Fourier series which doesn't vanish also has a reciprocal which is convergent, and which is provable in three lines if you look at it the right way but it is a hard thing to prove if you try to do it by brute force. I thought that was very attractive.

The use of abstract techniques from functional analysis, there was this proof that Wermer showed of the Stone-Weierstrass theorem. This proof says just look at the extreme points of the unit ball of the annihilator and they have got to be points so you win. It is just coming from this basic fact of the compactness of the unit ball of the dual and the fact that weak-star compact convex sets have extreme points – I love that. So I would have used that kind of approach tackling a problem about sums of algebras.

I notice things. When you are younger, you notice things and I tucked them away for future reference.

I like it when things mix together as well. I would be on the look-out for connections. The other thing is that it never hurts to know things and it is always a good idea to

keep learning. My favourite results took years to obtain, and the final step usually followed digesting some new theory.

PM: *Which are your favourite results?*

AOF: I have a soft spot for the generalised Walsh-Lebesgue Theorem (research paper 7 [5]). That was the first result I really liked. It has a kind of polished perfection. Simple statement, deep proof, involving Caratheodory's beautiful theory of prime ends and Glicksberg's generalised F.&M. Riesz Theorem, on top of the original Walsh-Lebesgue. I proved it not long after moving to UCLA, and it was my first *Meisterstück*: I felt I was no longer a journeyman, but finally a master craftsman, like my father and grandfathers. It is published in a minor journal, because Wermer had just been appointed to their editorial board and asked me if he could give it to them, but it is good enough for anywhere.

Hard-won and beautiful were the theorem with Marshall on sums of algebras (paper 28), the theorem with Preskenis on the algebra generated by two plane homeomorphisms (paper 41), and the theorem with Alejandro Sanabria-Garcia on De Paepe's disc (paper 63). The papers on capacities, on derivations, and on reversibility represent stages in ongoing campaigns, open-ended results, and have attracted other workers abroad, but there is a particular satisfaction when you kill off a topic with a single blow, and I have a good few papers that do that. Some other efforts that have attracted little notice have potential for development. A young person could do worse than study my least-cited papers.

PM: *Thank you very much indeed for your time Tony.*

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