Reports of Sponsored Meetings

Reports received of sponsored meetings held in 2024:

Groups in Galway 2024
16–17 May 2024, University of Galway

The 2024 instalment of the series of meetings 'Groups in Galway' took place at the University of Galway on 16–17 May 2024. The meeting was organised by Angela Carnevale, Joshua Maglione and Tobias Rossmann. It was supported by Athena SWAN Ireland, by the de Brún Centre for Mathematics, by the Irish Mathematical Society and by the Office of the Registrar and Deputy President of the University of Galway. There were 8 invited speakers and over 30 participants.

The talks covered a wide range of topics in contemporary group theory and related fields and were spread over three sessions.

The invited speakers were:

(1) Anna Giordano Bruno (University of Udine)
(2) Alex Evetts (University of Manchester)
(3) Itay Glazer (University of Oxford)
(4) Waltraud Lederle (Dresden University of Technology)
(5) Mireille Soergel (Max Planck Institute for Mathematics in the Sciences)
(6) Mima Stanojkovski (University of Trento)
(7) Matteo Vannacci (University of the Basque Country)
(8) Andoni Zozaya (University of Ljubljana)

Titles and abstracts:

- Anna Giordano Bruno: *A brief history and recent advances in the theory of characterized subgroups of the circle group.*
  A subgroup H of the circle group $\mathbb{T}$ is said to be *characterized* by a sequence of integers $\mathbf{u} = (u_n)_{n\in\mathbb{N}}$ if $H = \{x \in \mathbb{T} : u_n x \to 0\}$. The first part of the talk discusses characterized subgroups of $\mathbb{T}$ and their relevance in several areas of Mathematics where the behaviour of the sequence $(u_n x)_{n\in\mathbb{N}}$ as above is studied, such as Topological Algebra (topologically torsion elements and characterized

subgroups), Harmonic Analysis (sets of convergence of trigonometric series, thin sets) and Number Theory (uniform distribution of sequences).

Recently, generalizations of the notion of a characterized subgroup of $\mathbb{T}$ were introduced, based on weaker notions of convergence, starting from statistical convergence and ending with $\mathcal{I}$-convergence for an ideal $\mathcal{I}$ of $\mathbb{N}$, due to Cartan. A sequence $(y_n)_{n\in\mathbb{N}}$ in $\mathbb{T}$ is said to *$\mathcal{I}$-converge* to a point $y \in \mathbb{T}$, denoted by $y_n \xrightarrow{\mathcal{I}} y$, if $\{n \in \mathbb{N} : y_n \notin U\} \in \mathcal{I}$ for every neighborhood U of y in $\mathbb{T}$. A subgroup H of the circle group $\mathbb{T}$ is said to be *$\mathcal{I}$-characterized* with respect to $\mathcal{I}$ by a sequence of integers $\mathbf{u} = (u_n)_{n\in\mathbb{N}}$ if

$$H = \{x \in \mathbb{T} : u_n x \xrightarrow{\mathcal{I}} 0\}.$$

The second part of the presentation proposes an overview on the results obtained on this new kind of characterized subgroups, with special emphasis on $\mathcal{I}$-characterized subgroups of $\mathbb{T}$.
Based on a joint work with D. Dikranjan, R. Di Santo and H. Weber.

- Alex Evetts: *Twisted conjugacy growth of virtually nilpotent groups.*
The conjugacy growth function of a finitely generated group is a variation of the standard growth function, counting the number of conjugacy classes intersecting the n-ball in the Cayley graph. The asymptotic behaviour is not a commensurability invariant in general, but the conjugacy growth of finite extensions can be understood via the twisted conjugacy growth function, counting automorphism-twisted conjugacy classes. I will discuss what is known about the asymptotic and formal power series behaviour of (twisted) conjugacy growth, in particular some relatively recent results for certain groups of polynomial growth (i.e. virtually nilpotent groups).

- Itay Glazer: *Fourier and small ball estimates for word maps on unitary groups.*
Let $w(x, y)$ be a word in a free group. For any group G, w induces a word map $w : G^2 \to G$. For example, the commutator word $w = xyx^{-1}y^{-1}$ induces the commutator map. If G is finite, one can ask what is the probability that $w(g, h)$ is equal to the identity element e, for a pair (g, h) of elements in G, chosen independently at random. In the setting of finite simple groups, Larsen and Shalev showed there exists $\epsilon(w) > 0$ (depending only on w), such that the probability that $w(g, h) = e$ is smaller than $|G|^{-\epsilon(w)}$, whenever G is large enough (depending on w). In this talk, I will discuss analogous questions for compact groups, with a focus on the family of unitary groups; For example, given a word w, and given two independent Haar-random $n \times n$ unitary matrices A and B, what is the probability that $w(A, B)$ is contained in a small ball around the identity matrix?
Based on a joint work with Nir Avni and Michael Larsen.

- Waltraud Lederle: *Boomerang subgroups.*
Given a locally compact group, its set of closed subgroups can be endowed with a compact, Hausdorff topology. With this topology, it is called the Chabauty space of the group. Every group acts on its Chabauty space via conjugation. This action has connections to rigidity theory, Margulis' normal subgroup theorem and measure preserving actions of the group via so-called Invariant Random Subgroups (IRS). I will give a gentle introduction into Chabauty spaces and IRS and state a few classical results. I will define boomerang subgroups and explain how special cases of the classical results can be proven via them.
Based on joint work with Yair Glasner.

- Mireille Soergel: *Dyer groups: Coxeter groups, right-angled Artin groups and more.*
  Dyer groups are a family encompassing both Coxeter groups and right-angled Artin groups. Each of these two classes of groups have natural piecewise Euclidean CAT(0) spaces associated to them: the Davis-Moussong complex for Coxeter groups and the Salvetti complex for right-angled Artin groups. In this talk I will introduce Dyer groups and give some of their properties.

- Mima Stanojkovski: *Studying p-groups via their Pfaffians: isomorphism testing and the PORC conjecture.*
  Given a field K, to each alternating $n \times n$ matrix of linear forms in $K[y_1, \ldots, y_d]$ one can associate a group scheme G over K. In particular, when K is the field of rationals and F is the field of p elements, the F-points $\mathrm{G}(F)$ of G form a group of order p^{n+d} and so, as p varies, one obtains an infinite family of p-groups from G. In this talk, I will present joint work with Josh Maglione and Christopher Voll, as well as ongoing work with Eamonn O'Brien, on the geometric study of automorphisms and isomorphism types of groups associated to small values of the parameters n and d. I will also explain the implications of our work in connection to claims made around Higman's famous PORC conjecture.

- Matteo Vannacci: *Profinite groups of finite probabilistic virtual rank.*
  A profinite group G carries naturally the structure of a probability space, namely with respect to its normalised Haar measure. We study the probability $Q(G, k)$ that k Haar-random elements generate an open subgroup in the profinite group G. In particular, in this talk I will introduce the probabilistic virtual rank $\mathrm{pvr}(G)$ of G; that is, the smallest k such that $Q(G, k) = 1$. We will discuss some key theorems and open problems about random generation in profinite groups, with a view toward finite direct products of hereditarily just infinite profinite groups. Classic examples of the latter type of groups are semisimple algebraic groups over non-archimedean local fields. This is joint work with Benjamin Klopsch and Davide Veronelli.

- Andoni Zozaya: *Linearity of compact analytic groups over domains of characteristic zero.*
  A p-adic analytic group is a topological group that is endowed with an analytic manifold structure over $\mathbb{Z}_p$, the ring of p-adic integers. This definition can be extended by considering the manifold structure over more general pro-p domains, such as the power series rings $\mathbb{Z}_p[[t_1, \ldots, t_m]]$ or $\mathbb{F}_p[[t_1, \ldots, t_m]]$ (where $\mathbb{F}_p$ denotes the finite field of p elements).

  Lazard established already in the 1960s that compact p-adic analytic groups are linear, as they can be embedded as a closed subgroup within the group of invertible matrices over $\mathbb{Z}_p$. Nonetheless, the question of the linearity of analytic groups over more general domains remains unsolved.

  In this talk, we shed some light to this question by proving that when the coefficient ring is of characteristic zero, every compact analytic group is linear. We will provide background on the problem and outline the strategy of our argument. Joint with M. Casals-Ruiz.

The conference website is to be found at `https://groupsingalway.github.io/posts/GiG2024`.

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22nd Galway Topology Colloquium
4–5 June 2024, University of Galway

The 22nd Galway Topology Colloquium took place at the University of Galway on 4–5 June 2024. It was organised by Aisling McCluskey and Nina Snigireva (both from University of Galway).

The Colloquium Series was established in 1997 by Aisling McCluskey and Paul Gartside and rotates annually between the centres of topological research in Ireland and the UK (apart from the Covid period). The colloquia provide postgraduates and early career researchers in topology, as well as seasoned academics, with the opportunity to share their research in a friendly, informal and supportive environment. Topology is interpreted broadly and includes set-theoretic topology, algebraic topology, continua theory, topological dynamics, as well as cross fertilisation between topology and category theory, order theory, metric space theory, and analysis.

There were a total of seven invited speakers:

- Paul Bankston (Marquette University, US)
  Betweenness and Equidistance in Hyperspaces.
- K. P. Hart (TU Delft)
  Many subalgebras of $\mathcal{P}(\omega)/fin$: A tale of mass murder and mayhem.
- John C. Mayer (University of Alabama at Birminghmam, US)
  Complex Dynamics: Polynomials, Julia Sets, Parameter Spaces, and Laminations.
- Anca Mustata (University College Cork)
  Families of manifolds with large symmetry groups.
- Richard Smith (University College Dublin)
  de Leeuw representations of functionals on Lipschitz spaces.
- Filip Strobin (Łódź University of Technology, Poland)
  Rate of convergence in the deterministic version of the chaos game algorithm.
- Stephen Watson (York University, Toronto)
  On the existence of Nash equilibrium.

The following speakers also contributed talks::

- Daron Anderson
  Non-Block Points in Hereditarily Decomposable Continua.

- Christopher Boyd (University College Dublin)
  Order Continuous Polynomials.
- Robin Knight (University of Oxford)
- Simo S. Mthethwa (University of KwaZulu-Natal, South Africa)
  A few points in pointfree topology.

Abstracts of Invited Talks:

- Paul Bankston: *Betweenness and Equidistance in Hyperspaces.*
  We explore what it means when one compact set lies between - or is equidistant from - two others, in the context of metric spaces. We are also interested in notions of convexity that arise from these considerations.

- Klaas Pieter Hart: *Many subalgebras of $\mathcal{P}(\omega)/fin$: A tale of mass murder and mayhem.*
  In answer to a question on MathOverflow we show that the Boolean algebra $\mathcal{P}(\omega)/fin$ contains a family $\{\mathcal{B}_X : X \subseteq \mathfrak{c}\}$ of subalgebras with the property that $X \subseteq Y$ implies $\mathcal{B}_Y$ is a subalgebra of $\mathcal{B}_X$ and if $X \not\subseteq Y$ then $\mathcal{B}_Y$ is not embeddable into $\mathcal{B}_X$. The proof proceeds by Stone duality and the construction of a suitable family of separable zero-dimensional compact spaces.

- John Mayer: *Complex Dynamics: Polynomials, Julia Sets, Parameter Spaces, and Laminations.*
  Laminations are a combinatorial and topological way to study connected Julia sets of polynomials. While each locally connected Julia set has a corresponding lamination, laminations also give information about the structure of the parameter space of degree $d \geq 2$ polynomials with connected Julia sets. A d-invariant lamination of the unit disc consists of a closed collection of chords, called leaves, which meet at most at their endpoints, and which is forward and backward invariant under the angle-d-tupling map on the unit circle. Of particular interest are leaves in a lamination which are periodic, return for the first time by the identity, and whose endpoints are in different orbits. Such leaves play an important and understood role in the parameter space of quadratic polynomials and in the parameter spaces of unicritical higher degree polynomials, but more study is needed in the more general case of multiple criticality. Here we focus on the first case where there are open questions about the laminations: the angle-tripling map corresponding to degree 3 polynomials with connected Julia set.
  Coauthors: Brittany E. Burdette and Thomas C. Sirna.

- Anca Mustata: *Families of manifolds with large symmetry groups*
  In this talk we discuss families of complex projective varieties with relatively large groups of symmetry, which can be found as moduli spaces of objects in highly symmetric complex projective hypersurfaces. We discuss special families of $(n-3)$-dimensional complex varieties whose automorphism groups lie inside the $(n+1)$-th symmetric group. A particular case is the Wiman-Edge pencil of genus 6 complex projective curves. First found in a paper in 1895 by A. Wiman, its modular interpretation was first found by Ph. Candelas, X. de la Ossa, B. van Geemen, D. van Straten in 2012 and explained by Zagier (2014) and I. Dolgachev, B. Farb, E. Looijenga (2018), who proved that every smooth projective curve of genus 6 endowed with a faithful A5-action is equivariantly isomorphic with a member of this pencil.

- Richard Smith: *de Leeuw representations of functionals on Lipschitz spaces.*
Let $\mathrm{Lip}_0(M)$ be the Banach space of Lipschitz functions on a complete metric space (M,d) that vanish at a point $0 \in M$. This has an isometric predual $\mathcal{F}(M) \subset \mathrm{Lip}_0(M)^*$, called the Lipschitz-free (hereafter free) space over M. Free spaces are at the interface between functional analysis, metric geometry and optimal transport theory. They are the canonical way to express metric spaces in functional analytic terms, analogously to how compact Hausdorff spaces can be expressed using $C(K)$-spaces.

  We still have a quite poor understanding of the spaces $\mathcal{F}(M)$ and (even more so) their biduals $\mathrm{Lip}_0(M)^*$. Their structure can be probed using the 'de Leeuw transform', which yields representations of each functional on the Lipschitz space $\mathrm{Lip}_0(M)$ in the form of (non-unique) measures on the Stone-Čech compactification $\beta\widetilde{M}$ of $\widetilde{M} := \{(x,y) \in M \times M \,:\, x \neq y\}$.

  In this talk we introduce the above and show how topological concepts such as the uniform compactification and 'Lipschitz realcompactification' of (M,d), can be used to study de Leuuw representations of elements of $\mathcal{F}(M)$ and $\mathrm{Lip}_0(M)^*$ and thus shed light on the structure of these spaces. Along the way we introduce a 'metric bidual' of (M,d), whose relationship with (M,d) is analogous to the relationship between a Banach space and its bidual.

  This is joint work with Ramón Aliaga (Universitat Politècnica de València) and Eva Pernecká (Czech Technical University, Prague).

- Filip Strobin: *Rate of convergence in the deterministic version of the chaos game algorithm.*
The validity of the classical chaos game algorithm for generating images of attractors of contractive iterated function systems can be explained by the fact that, with probability 1, a randomly chosen sequence from a given finite alphabet is disjunctive, meaning that it contains all finite words from that alphabet as its subwords. In particular, given a disjunctive sequence, the generated orbit will approximate the attractor. During my talk I will explain how to measure the rate of convergence of orbits to the attractors and show that additional properties of disjunctive sequences give some control over that rate. On the other hand, I will show that a typical (in the sense of Baire's category and even porosity) disjunctive sequence does not give any control over the rate of convergence. Finally, I will present the result which shows that the situation can be completely different from the probabilistic point of view - in some cases, with probability 1, the rate of convergence of a randomly chosen driver is controlled by the dimension of the invariant measure.

  Results related to the deterministic chaos game is joint work with Krzysztof Leśniak and Nina Snigireva, and can be found in K. Leśniak, N. Snigireva, F. Strobin, *Topological prevalence of variable speed of convergence in the deterministic chaos game*, Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat. 118, 157 (2024) and in K. Leśniak, N. Snigireva, F. Strobin, *Rate of convergence in the disjunctive chaos game algorithm*, Chaos 32 (2022), no. 1, Paper No. 013110.

  Results related to the probabilistic chaos game can be found in B. Bárány, N. Jurga, I. Kolossváry, *On the convergence rate of the chaos game*, Int. Math. Res. Not. 2023 (2023), no. 5, 4456-4500.

- Stephen Watson: *On the existence of Nash equilibrium.*
Nash equilibrium is regarded as one of the most important notions in Game Theory. The concept dates back to at least Cournout. However, its current

formalization is due to Nash, whose original proof, given in 1950, relies on Kakutani's fixed point theorem. One year later, Nash gave a different proof, which uses Brouwer's fixed point theorem.

The self-contained proof here makes no use of fixed point theorems. Our proof can be split in two parts. The first part introduces two new notions: root function and distributed equilibrium. A root function is a map from the set of mixed strategy profiles to the set of pure strategy profiles. A distributed equilibrium is a subset of mixed strategy profiles that generalizes Nash equilibrium. In the second part, elaborating an argument used by McLennan and Tourky, we show that arbitrarily small distributed equilibria always exist. By means of compactness, we obtain the existence of a Nash equilibrium.
Joint work with D. Carpentiere.

The conference website is to be found at `https://maths.nuigalway.ie/galwaytopology/`.

Report by Aisling McCluskey and Nina Snigireva, University of Galway
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CETL-MSOR 2024
28–30 August 2024, University of Limerick

CETL-MSOR 2024 was hosted in the University of Limerick on the 28$^{\text{th}}$, 29$^{\text{th}}$ and 30$^{\text{th}}$ of August this year. This annual conference is a meeting of practitioners of the teaching, learning and support of mathematics, statistics and operations research in higher education. This was only the second time this conference was held outside the UK. The conference themes this year were:

- Linking research and practice in mathematics and statistics education in Higher Education – opportunities and challenges;
- Teaching mathematics for mathematics specialist and non-mathematics specialist groups;
- The changing nature of mathematics and statistics learning support;
- Exploring the affective domain in third level mathematics and statistics education.

Eighty delegates from the UK, the US, Ireland and Eastern Europe were in attendance. As part of the conference schedule, the delegates were given a tour of UL Glucksman

Library which homes a fascinating array of ancient and very rare mathematical texts. Professor Kenneth Stanton, the executive dean of the Faculty of Science and Engineering, officially opened the conference.

Pictured below are our keynotes speakers with Dr Olivia Fitzmaurice, Chair of the 2024 CETL-MSOR conference organisation committee.

Dr Eabhnat Ní Fhloinn (DCU), Dr Olivia Fitzmaurice (UL - conference chair), Dr Joe Kyle (Birmingham University), Dr Rafael De Andrade Moralis (Maynooth University).

Our keynote speakers were:

Dr Eabhnat Ní Fhloinn (Dublin City University):
Mathematics Learning Support in Ireland: Do we know it Inside Out?
Abstract: It is over 20 years since the first Mathematics Learning Support (MLS) Centre opened in the University of Limerick, where we now find ourselves celebrating CETL-MSOR 2024. During this time, MLS has expanded and become viewed as a mainstream support service in many Higher Education Institutes. The Irish Mathematics Learning Support Network (IMLSN) has played a pivotal role in this development, and in bringing together practitioners and researchers from around the country, much as other similar networks have done in the UK, Scotland and Germany. In this talk, we consider the historical challenges faced by MLS in Ireland, look at what we learned from these, and explore any new challenges facing us in the coming years. We ask the question - after more than twenty years of MLS in Ireland, do we know it inside out?

Dr Rafael De Andrade Moralis (Maynooth University):
Notes and Tricks for Teaching Statistics using Music and Magic.
Abstract: In this talk, I will share my recent experience using musical parodies and magic tricks to teach different statistical concepts. I will draw parallels between my lecturing experience in Brazil and in Ireland, and discuss how I use general pedagogy and active methodologies to encourage student participation. I will also discuss successful approaches, as well as other approaches still under development. I will showcase some of these activities in the context of explaining the concepts of conditional probability, p-values, and hypothesis tests. Finally, I will present the tools and equipment I currently use to produce music videos to teach statistics and give tips on what I think has helped improve the quality of the materials I have been producing.

Dr Joe Kyle (Birmingham University):
Beyond the Grave Morrice.
Dr Kyle gave the closing plenary presentation in which he discussed developments in mathematics education, mainly the use of AI, and concluded with insights gained over the course of conference.
Abstract: Casting a glance backwards as well as looking into the future (as far as that is possible), this talk will take upon itself the task of responding to and reacting to developments reported this year at the Limerick conference. As we struggle to harness the power of generative AI (or is it we who are being harnessed?) we look back to tried and tested axioms that may guide us on the new adventures ahead. And, as problem-solving is at the heart of mathematics, and problems are at the heart of problem-solving, there may be the odd puzzle to keep us all awake.

Dr Ciarán Mac an Bhaird was the recipient of 2024 international award 'The Lawson-Croft Award for Outstanding Achievement in Mathematics and Statistics Support'. Dr Mac an Bhaird is pictured with Professor Michael Grove, Deputy Pro-Vice-Chancellor, University of Birmingham who made the announcement at the conference.

We would sincerely like the thank our sponsors who ensured the conference was a success: EPI*STEM, the National Centre for STEM Education; the Centre for Transformative Learning (UL); The President's Office (UL); The Department of Mathematics and Statistics (UL); and the Irish Mathematical Society (IMS).

The Local Organisation Committee comprised: Dr Olivia Fitzmaurice - Chairperson, Dr Richard Walsh, Dr Aoife Guerin, Dr Patrick Johnson, Dr Niamh O'Meara, Prof. John O'Donoghue.

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