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A. B. Sossinsky: Knots, Links and Their Invariants: An Elementary Course in Contemporary Knot Theory, American Mathematical Society, 2023.

ISBN:978-1-4704-7151-4, USD 59, 128+xvii pp.

REVIEWED BY ASHLEIGH WILCOX

Despite its small size, the book holds an abundance of information and provides an introduction to knot theory as a series of thirteen lectures. The author assumes no prerequisites of the reader, other than basic plane and spatial Euclidean geometry. Having never previously studied topology, I was interested to see whether this book would be accessible for myself, and therefore the readership: undergraduate and graduate students interested in knot theory.

Originally, Sossinsky presented this course online to students across the world and then published a small print run of the lecture notes. This book is a revised version of those lecture notes, complete with numerous illustrations and exercises for the reader to complete, although without solutions.

This book provides an excellent framework for an introductory course into knot theory. It contains the important theorems of the last 40 years, with proofs using mostly elementary methods. All necessary preliminary material is included.

The first eleven lectures are essentially self-contained, with the only references being to tables contained in [1] and [2]. The final two chapters provide a rather brief overview of the history of knot theory and introduce further important topics which an interested reader could explore after the course.

Chapter 1 discusses what will be learnt and how examples will reappear throughout the course. This drip-feeding of information is very helpful, as it allows you to see the reasoning behind the theory you are introduced to, and their importance becomes clear further into the course. As well as useful and well-thought out diagrams to support explanations and understanding, there are also intuitive descriptions. This ensures the book is accessible and largely well-explained. Despite being an introductory course, the author manages to convey the topic in a way that encourages you to pursue the topic further and it makes you want to learn more about knot theory, especially with the final two chapters giving a brief introduction to other topics and the mathematicians that were pivotal in the growing of this topic.

A brief insight into some chapters of the book is as follows.

Chapters $1-3$ provide an introduction into the main elements of knot theory, covering Reidemeister moves, the Conway polynomial and the arithmetic of knots.

Chapters $4-6$ contain interesting theorems and definitions and facts.

Chapter 6 also has information on things that the reader could further explore, for example the Dowker-Thistlewaite code.

Chapter 7 relates theory of braids to where their applications appear.

The final lecture provides a brief history of knot theory and also contains images of the main contributors to the subject.

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At the end of each chapter there is a number of exercises. These exercises are challenging, but allow for a deep understanding of the material. Some of the exercises ask for proofs to theorems introduced in the lectures, and proofs for how some lemmas imply theorems. This shows how the material interacts. This layout provides an interesting flow of the content and shows how much of the course content can be derived through elementary knowledge.

There are some areas of the book which could benefit from some further explanations, such as how to calculate $\nabla(L)$, and how to calculate the link number. A course based on the material in this book would provide a good framework and introduction to knot theory. The historical account in the final chapter is divided in sections, so if the book is used to support a taught course, the history could be interwoven into the relevant sections of the theory. This could be especially useful for an undergraduate audience, showing when and by whom the theory was introduced. It could also be suitable as a self-study guide or textbook, although it may be beneficial to have as a taught course, especially as a number of proofs to theorems, lemmas and remarks are left as exercises for the reader. However, the course is very accessible for the recommended readership and an interested reader has much of the information necessary to build knowledge to understand and learn the content.

In summary, the author has created a very useful and concise book that provides an interesting introduction to some of the main results of knot theory such as the Alexander-Conway knot polynomial, the Jones polynomial and the Vassiliev invariants.

References

[1] S. Chmutov, S. Duzhin, and J. Mostovoy: *Introduction to Vassiliev knot invariants*, Cambridge University Press, 2012.

[2] D Rolfsen: *Knots and links*, Mathematics Lecture Series, No. 7, Publish or Perish, Inc., Berkeley, Calif., 1976.

Ashleigh Wilcox is a second-year PhD student and Graduate Teaching Assistant at the University of Leicester. She graduated with a Mathematics BSc in 2022. Her main mathematical interests are in Number Theory, with a current research project: a systematic approach to solving Diophantine equations. She is passionate about outreach and inclusion in mathematics and volunteers as a STEM Ambassador and is the University of Leicester representative for the Piscopia Initiative.

School of Computing and Mathematical Sciences, University of Leicester
E-mail address: `amw64@leicester.ac.uk`