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Des MacHale: Lateral Solutions to Mathematical Problems, A K Peters/CRC Press. Taylor & Francis Group, 2023. ISBN: 978-100-3341-46-8, GBP 26.99, 123+xiv pp.

REVIEWED BY PETER LYNCH

This book is a rich source of delightful puzzles in school and first-year undergraduate mathematics: number theory, geometry, trigonometry, calculus, probability and logic, as well as miscellaneous other topics. The problems often look daunting, but solutions can usually be found by means of lateral reasoning. Lateral thinking (LT) is a method of solving problems by adopting a new perspective.

The book will be of interest to students and also to teachers and lecturers seeking new material to spark the imaginations of their students. There are ten problems in each of the twelve sections, giving a total of 120 problems. These vary greatly in difficulty. In most cases, some clever mental gymnastics can reduce an apparently intractable problem to a simple calculation or line of logic. In a smaller number, the solution is tricky even after the answer is presented.

Problems in Section 1 are on Number Theory, beginning with this: If you multiply all the primes less than one million together, what is the final digit of your answer? A moment's reflection gives the answer but, as for all the problems, the solution is augmented by suggestions for "further investigation", which enrich the text.

Another easy problem asks for the final digit of 2^{999}? While the line of reasoning is correct, the answer 6 on p. 35 is not. The number has 301 decimal digits, ending in 688. Problem 1.8, to determine the sum of the first n Fibonacci numbers, is a clear and simple application of LT. Likewise, to show that $(n!)^2$ divides $(2n)!$, which is easily done by some "thinking outside the box".

Problem 2.9, a variation on a theme of Goldbach, is easy and is amusing to solve. The last problem in Section 2 is to show that the sum of reciprocals of the first n primes is never an integer. This is elementary but subtle and requires lateral imagination.

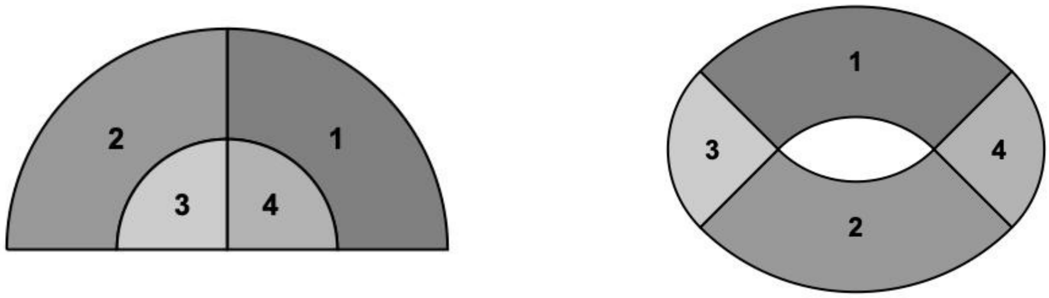

FIGURE 1. Rearranging a semicircle (left) to form an oval (right).

In Section 3, on Geometry, one problem requires us to cut a semicircle into four pieces and reassemble them into an ellipse. What we are not told is that the solution may

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have a hole in the middle (Figure 1). Is this a cheat, or a legitimate solution? I suppose it depends on the latitude of the lateral thinking! Moreover, the outer boundary is not an ellipse but a set of four circular arcs.

Problem 3.7 asks for the area of the smaller square in Figure 2, given the area of the larger one. It is a delightful puzzle and the simple solution, requiring no calculation, should produce a warm glow of satisfaction. Spoiler alert: no peeping ahead to Figure 3!

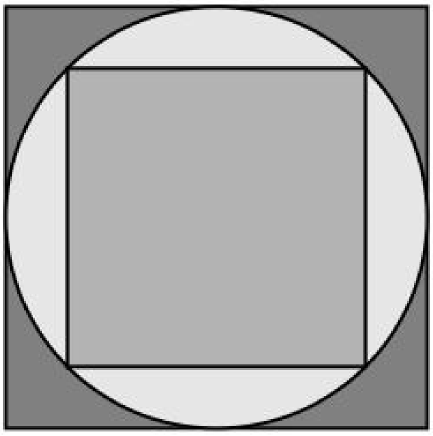

FIGURE 2. Circle with circumscribed and inscribed squares.

How many observations of the position of a planet are needed to determine its orbit? Three suffice for a circular orbit, so we might guess four for an ellipse. But we need the direction as well as the extent of the broken circular symmetry, so five observations are required. The solution of Problem 3.10 has a diagram confirming this.

On Trigonometry, Problem 4.1 is solved by noting that $\tan 45^\circ = 1$ and using the addition formula for tangents. This is described as a "lovely lateral twist". Another problem asks for a triangle such that the tangents of all three angles are positive integers. The (unique) answer is quite surprising. Problem 4.3, on an infinite sum of inverse sines of inverse radicals, looks hopeless. I got nowhere by staring at it. Readers will need inspiration, whether lateral, dorsal, ventral or even sagittal, to solve this without a peep at page 52.

Probability problems can be quite counter-intuitive, and ambiguous if not carefully formulated. Problem 5.4 illustrates this: a thin rod is broken at random into three pieces. What is the probability that the pieces can be used to form a triangle? There is a subtlety here: how are the break-points chosen? (1) By choosing two random break-points on the rod or (2) by breaking once, then breaking the longer piece. The solution given is for the first method but the second seems more natural. The probability depends on the method of breaking the rod. This is analogous to Bertrand's paradox, which reappears in Problem 5.9.

Problem 5.7 asks: what is the probability that your father and mother have birthdays six months or less apart? I am ashamed to say that my initial guess for this simple problem was wrong. Most readers should not fall so easily into the trap.

Combinatorial problems can be fiendishly difficult, and you will need all your LT skills. Problem 6.9 where we must show that a set with precisely n elements has exactly 2^n subsets, is described by the author as "one of the most important facts in mathematics, central to a whole lot of theory and applications." Three distinct proofs are given, and also a discussion of mathematical induction.

Section 7 (on Dissections) has several problems on cutting up and rearranging polygons. Some are simple, some more challenging. As an example: Can you cut a 30×30 square piece of carpet to cover a 25×36 floor exactly? Surprisingly, a single cut suffices (provided it is the right one!).

Section 8, on "Matchsticks and Coins", has several minor but irksome errors. The figures are of poor quality and, in some cases, misleading. In Problem 8.2, the figure already shows the solution. In 8.5 there are 14 matches in the problem (p. 19) but 15 in the solution (p. 80). In 8.6, the added match is not the same length as the originals.

In Section 9, on logic, warm-up questions such as *Is "no" the answer to this question?* will sharpen up the reasoning. Several "old chestnuts" appear, but they are probably unfamiliar to younger readers. One problem I had not seen before (Problem 9.6) is intriguing: *You are given two ropes, each of which takes exactly 60 minutes to burn. They are made of different materials and they burn at different rates and inconsistently. How can you measure 45 minutes exactly by burning the ropes?* Here you really need to think laterally. It is worthwhile taking a little time on this, to experience an Aha-moment.

In Section 10, on Maxima and Minima, some problems that would normally be solved by calculus are tackled without it using LT. Section 11 has some harder questions, leading up to the Painter's Paradox, dealing with a finite volume having infinite surface area. Problem 11.4 asks for a proof of the Arithmetic Mean-Geometric Mean inequality. The proof presented is attributed to the great Hungarian-American mathematician George Pólya. A typo in Problem 11.6 may confuse readers.

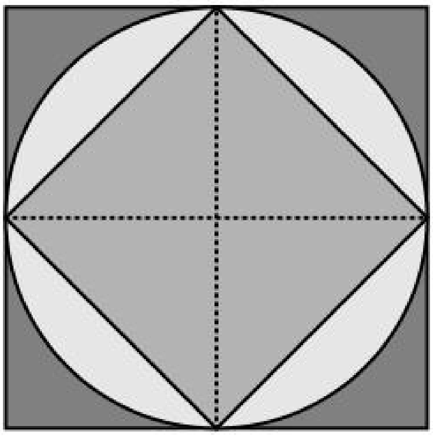

FIGURE 3. Solution to problem in Fig. 2.

Section 12 contains a miscellany of puzzles to round off the collection. The mnemonic rhyme for π has sixth word "remember" while the relevant digit of π is 9, and there are better *piems*, or poems about pi. In Problem 12.5, a quadratic is solved without completing the square, but I suspect that something is hidden here.

Summary. It is more than fifty years since LT was popularised by Edward De Bono's book *The Use of Lateral Thinking*, but the concepts and methods that he solidified and made practical are of lasting value. It is good to see these methods employed effectively to solve simple and not-so-simple mathematical problems. Despite some flaws, and the regretted absence of an index, this book is a nice collection of puzzles, problems, paradoxes and enigmas. It would also serve as an ideal gift for a mathematically-inclined friend.

Peter Lynch is emeritus professor in the School of Mathematics & Statistics, UCD. He is interested in all areas of mathematics and its history. He writes an occasional mathematics column in *The Irish Times* and has published three books of articles, *That's Maths, Vols. I, II and III.* A fourth volume is in preparation. His website is `https://maths.ucd.ie/~plynch` and his mathematical blog is at `http://thatsmaths.com`.

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