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Daniel Rosenthal, David Rosenthal and Peter Rosenthal: A Readable Introduction to Real Mathematics (2nd Edition), Springer, 2018. ISBN:978-3-030-80731-3, GBP 44.99, 218+xviii pp.

REVIEWED BY ROBIN HARTE

In this second edition of their delightful "readable introduction" the Rosenthal family have preserved the structure, Chapters 1-12, of the first edition, and meticulously gone through them picking out typos and minor infelicities, and then (the Operator Theory beginning to show through!) added two new chapters: on infinite series and then higher dimensional spaces, including norms and inner products.

Back in the early chapters, Customs Officials have searched the alleged prime number 100,000,559 and uncovered its cargo of prime factors 53, 223 and 8,461. As in the first edition, Chapters 1 to 3 introduce the natural numbers, the principle of mathematical induction, and then modular arithmetic. Chapters 4 and 5 are devoted to the fundamental theorem of arithmetic, prime factorisation, and then the theorems of Fermat and Wilson.

Chapter 6 is about the RSA method of "public key cryptography". Here the "recipient" publicly announces a number $N = pq$ which is the product of two very large and distinct prime numbers p and q, which are not revealed. Now a "message" is just a number $M < N$. To receive such messages the recipient announces another number E, the "encryptor", and asks the sender to compute, and send, the remainder R which M^E leaves on division by N. The recipient, wishing to determine M, must now find a decryptor D, such that for every $0 \leq L \leq N$, $L^{ED} \equiv L \bmod N$. With a little help from Fermat's theorem, it follows $R^D \equiv M \bmod N$.

Chapters 7 to 9 discuss the Euclidean algorithm, rational and irrational numbers, and then complex numbers. Chapter 10 is about cardinal numbers and infinite sets, and finally Chapters 11 to 12, "Euclidean plane geometry" and ruler-and-compass "constructibility", leading to (quadratic) "surds".

Towards the end, in the new Chapter 13, they offer the decomposition of a natural number as the product of a square free and a perfect square, which they deploy to prove that the reciprocals of the prime numbers combine to form a divergent series. Finally, in Chapter 14, they offer their introduction to norms and inner products.

This picky reviewer would have appreciated an appendix listing, in palatable form, axioms for the real numbers, and indeed "Euclidean plane geometry"; but the Rosenthals have between them produced a very fine, and very readable, introduction to "real" mathematics. Local readers should, as a supplement, also read the very beautiful notes ("Prime numbers", 2006 Course 4281 TCD tinyurl.com/4puvvr8p), from our own very much lamented T.G. Murphy.

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