

Problems

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I was asked the first problem during an interview at a bank in London several years ago.

Problem 67.1. Prove that there does not exist a differentiable function $f : \mathbb{R} \rightarrow \mathbb{R}$ that satisfies

$$f'(x) \geq 1 + [f(x)]^2$$

for each real number x .

I believe the second problem once appeared in The American Mathematical Monthly. I include it here because it is the only problem from Maynooth's current problem board that remains unsolved.

Problem 67.2. Suppose that x_1, x_2, \dots, x_n , where $n \geq 3$, are non-negative real numbers such that

$$x_1 + x_2 + \dots + x_n = 2$$

and

$$x_1x_2 + x_2x_3 + \dots + x_{n-2}x_{n-1} + x_{n-1}x_n = 1.$$

Find the maximum and minimum values of

$$x_1^2 + x_2^2 + \dots + x_n^2.$$

The last problem is apparently a classic. I first encountered it whilst working for the mathematics outreach project NRICH.

Problem 67.3. There are m gold coins divided unequally between n chests. An enormous queue of people are asked in turn to select a chest. Each member of the queue knows how many coins there are in each chest, and also knows the choice of those ahead in the queue who have selected already. In choosing a chest, each person considers the (possibly non-integer) number of gold coins he would receive were the coins in that chest to be shared equally amongst all those, including him, who have selected that chest so far. He then chooses the chest that maximises this number of coins. For example,

if there are three chests A , B , and C containing 3, 5, and 8 coins, then the first person in the queue selects C , the second selects B , the third selects C , the fourth selects A , and so forth.

After the m th person has chosen a chest, how many people have selected each chest? Express your answer in terms of the number of coins per chest. What more can be said about people's chest selections?

This problem is deliberately open ended, to encourage discussion and generalisation.

We invite readers to suggest their own problems, and to offer comments and solutions to past problems. In later issues we will publish solutions and acknowledge problem solvers. Please email submissions to imsproblems@gmail.com.

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