Abstracts of PhD Theses at Irish Universities 2009

Infinite Cycles in Boson Lattice Models

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This is an abstract of the PhD thesis *Infinite Cycles in Boson Lattice Models* written by Gerard G. Boland under the supervision of Joseph V. Pulé at the School of Mathematical Sciences, College of Engineering, Mathematical and Physical Sciences, University College Dublin and submitted in September 2009.

We study the relationship between long cycles and Bose–Einstein condensation (BEC) in the case of several models. A convenient expression for the density of particles on cycles of length q is obtained, in terms of q unsymmetrised particles coupled with a boson field.

Using this formulation we reproduce known results on the Ideal Bose Gas, Mean-Field and Perturbed Mean-Field Models, where the condensate density exactly equals the long cycle density. Then we consider the Infinite-Range-Hopping Bose–Hubbard Model:

$$H_V^{\mathrm{BH}} = \frac{1}{2V} \sum_{x,y=1}^{V} (a_x^* - a_y^*)(a_x - a_y) + \lambda \sum_{x=1}^{V} n_x(n_x - 1)$$

in two cases, first for $\lambda = +\infty$, otherwise known as the hard-core boson model; and secondly for λ finite, representing a finite on-site repulsion interaction.

For the hard-core case, we find we may disregard the hopping contribution of the q unsymmetrised particles, allowing us to calculate an exact expression for the density of particles on long cycles. It is shown that only the cycle of length one contributes to the cycle density. We conclude that while the existence of a non-zero long cycle density coincides with the occurrence of Bose–Einstein condensation, the respective densities are not necessarily equal.

For the case of a finite on-site repulsion, we obtain an expression for the cycle density involving the partition function for a Bose–Hubbard Hamiltonian with a single-site correction again by neglecting the q unsymmetrised hop. Inspired by the Approximating Hamiltonian method we conjecture a simplified expression for the short cycle density as a ratio of single-site partition functions. In the absence of condensation we prove that this simplification is exact and use it to show that in this case the long-cycle density vanishes. In the presence of condensation we can justify this simplification when a gauge-symmetry breaking term is introduced in the Hamiltonian. Assuming our conjecture is correct, we compare numerically the long-cycle density with the condensate and again find that though they coexist, in general they are not equal.

On Abelian Ideals in a Borel Subalgebra of a Complex Simple Lie Algebra

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This is an abstract of the PhD thesis *On Abelian Ideals in a Borel Subalgebra of a Complex Simple Lie Algebra* written by Patrick Browne under the supervision of Dr. John Burns at the Department of Mathematics, National University of Ireland, Galway and submitted in December 2008.

Let $\mathfrak{g}$ be a complex simple Lie algebra and $\mathfrak{b}$ a fixed Borel subalgebra of $\mathfrak{g}$. We construct all maximal (with respect to containment) abelian ideals of $\mathfrak{b}$ by a variety of methods each having their own merits. We also derive formulas for their dimensions. Then we give a new proof of Kostant's theorem on the dimension of an abelian ideal. Finally we apply our results to give new examples of Einstein solvmanifolds.

We now give a brief historical account of interest in this area. In 1945, A. Malcev [1] determined the commutative subgroups of maximum dimension in the semisimple complex Lie groups. The maximal dimension of these commutative subgroups coincides with the maximal dimension of a commutative subalgebra of $\mathfrak{g}$. The next development was Kostant's [2] paper published in 1965, where he gave a connection between Malchev's result and the maximal eigenvalue of the Laplacian acting on the exterior powers ($\bigwedge^k \mathfrak{g}$) of the adjoint representation. In 1998 Kostant reported on the results of Peterson that the number of abelian ideals in the fixed Borel subalgebra of $\mathfrak{g}$ is $2^{\mathrm{rank}(\mathfrak{g})}$, and this paper was the genesis of much of the recent interest in this area. In [3], Panyushev and Röhrle while studying the relationship between spherical nilpotent orbits and abelian ideals of $\mathfrak{b}$, constructed all maximal abelian ideals, with the aid of a computer program [4], and observed a bijection between them and the set of long simple roots. Our method does not require the use of computer calculations. Suter in [5] found the maximal dimension of a maximal abelian ideal using the affine Weyl group, in terms of certain Lie theoretic invariants and gave a uniform explanation of the one to one correspondence between the long simple roots and the maximal abelian ideals. In [6] Papi and Cellini gave formulas for the dimension of all maximal abelian ideals in $\mathfrak{b}$, similar to that of Suter. The new formulas in this thesis are simpler and different in nature.

The methods used in the thesis rely heavily on the theory of graded Lie algebras. We also give an alternative proof of Kostant's theroem that does not require the representation theory used by Kostant. Finally our results are used to construct new examples of non compact Einstein solvmanifolds.

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Explicit Small Classifying Spaces for a Range of Finitely Presented Infinite Groups

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This is an abstract of the PhD thesis *Explicit small classifying spaces for a range of finitely presented infinite groups* written by Maura Clancy under the supervision of Dr Graham Ellis at the School of Mathematics, Statistics and Applied Mathematics, National University of Ireland, Galway and submitted in February 2009.

While a classifying space B_G exists for any group G, in reality given a group presentation, finding a productive B_G is by no means trivial. This thesis unearths explicit small classifying spaces for a range of finitely presented infinite groups and uses these spaces to deduce homological information on the groups.

In Chapter 2 we derive formulae for the second integral homology of any Artin group for which the $K(\pi,1)$-conjecture is known to hold, and for the third integral homology of the braid group A_n and the affine braid group $\tilde{A}_n$. The derivation and proofs are based on the cellular chain complex $C_*(\tilde{X}_D)$, where $\tilde{X}_D$ is the universal cover of a classifying space B_G for the group $G \in \{A_n, \tilde{A}_n\}$. Chapter 3 defines *polytopal* groups, actions and classifying spaces. We prove that a group G is polytopal when G is the semi-direct product of two polytopal groups N and Q. We show that $\tilde{B}_N \times \tilde{B}_Q$ is the universal covering space of a polytopal classifying space $\tilde{B}_G$ for G, where $\tilde{B}_N$ (resp. $\tilde{B}_Q$) is the universal covering space of a polytopal classifying space B_N for N (resp. B_Q for Q). We further show that the cellular chain complex $C_*(\tilde{B}_G)$ can be obtained as the total complex of a double complex with $Dim(B_N)$ rows and $Dim(B_Q)$ columns, a fact alternatively proven by Thomas Brady in his paper "Free resolutions for semi-direct products". Chapter 4 centres on Bieberbach groups; we realise six of the ten 3-dimensional Bieberbach groups as semi-direct products $G = N \rtimes_\alpha Q$, where N is 2-dimensional Bieberbach and $Q = C_\infty$. This technique can be extended to determine, inductively, classifying spaces for higher dimensional Bieberbach groups. Chapter 5 introduces *twisted Artin groups* $\mathfrak{A}_{\vec{D}}$ and shows that in some cases there exists a polytopal classifying space whose t-dimensional cells are indexed by the finite type subsets, of size t, of the generating set S. We show that 3-generator twisted Artin groups of large type

admit a two-dimensional classifying space. Using star graph techniques we show that such classifying spaces are non-positively curved for standard Artin groups of large type. In Chapter 6 we conjecture that certain groups are quasi-lattice-ordered and then use a GAP routine to experimentally investigate the word-reversing algorithm.

The Geometry of the Space of Oriented Geodesics of Hyperbolic 3-Space

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This is an abstract of the PhD thesis *The Geometry of the Space of Oriented Geodesics of Hyperbolic 3-Space* written by Nikos Georgiou under the supervision of Dr. Brendan Guilfoyle at the Institute of Techonolgy Tralee and submitted in June 2009.

In this thesis we construct a Kähler structure $(\mathbb{J}, \Omega, \mathbb{G})$ on the space $\mathbb{L}(\mathbb{H}^3)$ of oriented geodesics of hyperbolic 3-space $\mathbb{H}^3$ and investigate its properties. We prove that $(\mathbb{L}(\mathbb{H}^3), \mathbb{J})$ is biholomorphic to $\mathbb{P}^1 \times \mathbb{P}^1 - \overline{\Delta}$, where $\overline{\Delta}$ is the reflected diagonal, and that the Kähler metric $\mathbb{G}$ is of neutral signature, conformally flat and scalar flat.

We establish that the identity component of the isometry group of the metric $\mathbb{G}$ on $\mathbb{L}(\mathbb{H}^3)$ is isomorphic to the identity component of the hyperbolic isometry group. We show that the geodesics of $\mathbb{G}$ correspond to ruled minimal surfaces in $\mathbb{H}^3$, which are totally geodesic iff the geodesics are null.

We then study 2-dimensional submanifolds of the space $\mathbb{L}(\mathbb{H}^3)$ of oriented geodesics of hyperbolic 3-space, endowed with the canonical neutral Kähler structure. Such a surface is Lagrangian iff there exists a surface in $\mathbb{H}^3$ orthogonal to the geodesics of Σ.

We prove that the induced metric on a Lagrangian surface in $\mathbb{L}(\mathbb{H}^3)$ has zero Gauss curvature iff the orthogonal surfaces in $\mathbb{H}^3$ are Weingarten: the eigenvalues of the second fundamental form are functionally related. We then classify the totally null surfaces in $\mathbb{L}(\mathbb{H}^3)$ and recover the well-known holomorphic constructions of flat and CMC 1 surfaces in $\mathbb{H}^3$.

Topics in Computer Assisted Finite Group Theory

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This is an abstract of the PhD thesis *Topics in Computer Assisted Finite Group Theory* written by Robert Heffernan under the supervision of Prof. Des MacHale at the School of Mathematical Sciences, University College Cork and submitted in April 2009.

The phrase 'computer assisted' in the title of this thesis refers not to 'computational group theory' in the usual sense—that is, the developing of algorithms and data structures to compute information about groups—but to the *use* of the computer, particularly the GAP [4] algebra system, as a tool. The thesis is motivated by the opinion that the Small Groups Library [3] is one of the most interesting and useful resources in the arsenal of the finite group theorist. This library contains computer descriptions of all finite groups of order less than 2000 (excepting the groups of order 1024) as well as many groups of larger order. In the thesis we consider several different problems in finite group theory; in each case our first avenue of inquiry will have been to search the Small Groups Library.

In the first chapter we consider the sum of the degrees of the irreducible characters of a finite group G, which we denote by $T(G)$. This has been studied by Berkovich and Zhmud [2, Chapter 11] and by Berkovich and Mann [1]. When p^n divides the order of G, where p is a prime and $n \leq 6$, we produce bounds for $T(G)$. We then produce bounds for $T(G)$ when G has at most 14 conjugacy classes.

In the second chapter we consider $\mathrm{Pr}(G)$, the probability that two group elements commute, and the function $f(G) = T(G)/|G|$ which can also be seen as a indicator of the commutativity of G. These and other such indicators of commutativity have been studied by various authors (see, for instance, [6, 7, 5]). We prove several 'threshold' results that link the structure of the group G to the value of $\mathrm{Pr}(G)$ or $f(G)$. For example: if G is a finite non-abelian group such that $\mathrm{Pr}(G) > \frac{7}{16}$, then G' is cyclic. Similarly, if G is a finite non-abelian group such that $f(G) > \frac{5}{8}$, then G' is cyclic. Moreover, in both cases $|G|$ is even and these results are best possible. We also briefly investigate some other functions related to $\mathrm{Pr}(G)$ and $f(G)$.

In the third chapter our focus switches to automorphisms of finite groups. We classify the finite abelian groups A such that $|A| =$

$|\mathrm{Aut}(A)|$. We then classify the finite abelian groups A such that $|A|_2 = |\mathrm{Aut}(A)|_2$.

In the fourth chapter we consider holomorphs of finite groups. If G is a group then the holomorph of G, $\mathrm{Hol}(G)$, is the group $G \rtimes \mathrm{Aut}(G)$ where the action of $\mathrm{Aut}(G)$ on G is natural. We determine all p-groups P of order p^n, where $n \leq 11$, such that P occurs as the holomorph of some finite group. We then prove that if $H \cong \mathrm{Hol}(G)$, where G is a finite group and H has odd order, then H has order at least 3^{13}. This bound is best possible.

In the fifth chapter we define the notion of a *failsafe cover* of a finite group G. This is a group H such that G embeds in H in such a way that if we remove any non-identity element h from H we can still find an isomorphic copy of G in H that does not involve the element h. We give some examples of such failsafe covers and find some failsafe covers of minimal order for small-order groups.

In the final chapter we describe some partial progress made on two further problems.

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Normalisers of Reflexive Operator Algebras

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This is an abstract of the PhD thesis *Normalisers of Reflexive Operator Algebras* written by Martin McGarvey under the supervision of Ivan Todorov at the Department of Pure Mathematics, Queen's University Belfast and submitted in September 2009.

If $\mathcal{A}$ is an operator algebra acting on a Hilbert space $\mathcal{H}$, a unitary operator U on $\mathcal{H}$ is called a unitary normaliser of $\mathcal{A}$ if

$$U\mathcal{A}U^* = \mathcal{A}.$$

Unitary normalisers of von Neumann algebras were first studied in the 1930s by Murray and von Neumann [3] where they used these operators to construct factors of type II and III.

Let $\mathcal{H}$ be a Hilbert space and $\mathcal{A}$ be a subalgebra of the algebra $\mathcal{B}(\mathcal{H})$ of all bounded linear operators on $\mathcal{H}$. A normaliser of $\mathcal{A}$ is an operator $T \in \mathcal{B}(\mathcal{H})$ such that $T^*\mathcal{A}T \subseteq \mathcal{A}$ and $T\mathcal{A}T^* \subseteq \mathcal{A}$. We let $N(\mathcal{A})$ denote the set of all normalisers of $\mathcal{A}$. It is obvious that for each operator algebra $\mathcal{A}$, we have $N(\mathcal{A}) = N(\mathcal{A}^*)$. Here $\mathcal{A}^* = \{A^* : A \in \mathcal{A}\}$ is the adjoint algebra of $\mathcal{A}$.

Question: Let $\mathcal{A}$ and $\mathcal{B}$ be reflexive operator algebras such that $N(\mathcal{A}) = N(\mathcal{B})$. Is it true that either $\mathcal{A} = \mathcal{B}$ or $\mathcal{A} = \mathcal{B}^*$?

If $\mathcal{A}$ and $\mathcal{B}$ are von Neumann algebras the answer to the above question is easily shown to be affirmative. The question becomes more interesting if we allow $\mathcal{A}$ and $\mathcal{B}$ to belong to various classes of non-selfadjoint operator algebras. The classes considered in [4], and in the present work, are subclasses of the family of CSL algebras. Two of the main results in [4] are that the question has an affirmative answer if $\mathcal{A}$ and $\mathcal{B}$ are either continuous nest algebras or totally atomic CSL algebras.

This thesis is dedicated almost entirely to the investigation of the above question. Given that it has an affirmative answer in the case $\mathcal{A}$ and $\mathcal{B}$ are continuous nest algebras, it seems natural to consider the question for larger classes of CSL algebras. In this thesis we are concerned with two such classes: the class of finite tensor products of continuous nest algebras and the class of nest subalgebras of von Neumann algebras. The latter were introduced by Gilfeather and Larson in [1] where they showed that any nest subalgebra of a von Neumann algebra containing a masa can be represented as a direct integral of nest algebras with respect to some direct integral decomposition of the underlying Hilbert space.

In this thesis we establish a result which relates the support of the tensor product of masa-bimodules to the Cartesian product of their supports. We introduce the concept of a normaliser orbit of an element of a reflexive operator algebra. We then show that the question has a positive answer whenever $\mathcal{A}$ is a finite length tensor product of continuous nest algebras and $\mathcal{B}$ is a CSL algebra.

We introduce doubly continuous nest subalgebras of von Neumann algebras and their invariant subspace lattices. We show that a doubly continuous multiplicity free CNvNSL is unitarily equivalent to the the tensor product of the Volterra nest with the projection lattice of a continuous masa. This allows us to prove that the question has a positive answer whenever $\mathcal{A}$ is a doubly continuous nest subalgebra of a von Neumann algebra and $\mathcal{B}$ is a CSL algebra.

We also investigate whether or not the question has a positive answer if only the sets of unitary normalisers of the CSL algebras are considered.

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Dihedral Codes

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This is an abstract of the PhD thesis *Dihedral Codes* written by Ian McLoughlin under the supervision of Professor Ted Hurley at the School of Mathematics, Statistics and Applied Mathematics, National University of Ireland, Galway and submitted in June 2009.

In this thesis we give new constructions of a number of extremal type II codes. Algebraic proofs are provided that the constructions do in fact yield the codes. The codes are constructed using group rings of which the underlying groups are dihedral. Type II codes are not cyclic, but the constructions here are similar to the constructions of cyclic codes from polynomials.

The first code we construct is the extended binary Golay code. It is constructed from a zero divisor in the group ring of the finite field with two elements and the dihedral group with twenty-four elements. We create a generator matrix of the code that is in standard form and is a reverse circulant generator matrix. The generator matrix generates the code as quasi cyclic of index two.

Algebraic proofs are given of the code's minimum distance, self-duality and doubly evenness. A list of twenty-three other zero divisors that we have found to generate the code is given. Trivial changes adapt the aforementioned algebraic proofs to any of these zero divisors. We also prove that the twenty-four zero divisors are the only ones of their form that will generate the code.

Next we construct the $(48, 24, 12)$ extremal type II code as a dihedral code. The new construction is similar to that of the extended Golay code. Again, proofs of the self-duality and doubly evenness are given. An algebraic proof of the minimum distance is achieved through the use of two different group ring matrices.

A number of different codes are then constructed, building on the first two constructions. We list some zero divisors that generate type II codes of lengths seventy-two and ninety-six. According to investigations by computer, these codes have minimum distances of twelve and sixteen respectively. No type II codes of each of these lengths are known that have greater respective minimum distances. Some techniques are detailed that vastly reduce the calculations involved in their analysis.

The constructions of the $(72, 36, 12)$ and $(96, 48, 16)$ codes are facilitated by the construction of extremal type II codes of all lengths a multiple of eight up to length forty. Type II codes only exist at lengths that are multiples of eight. Overall we showed that extremal type II codes can be constructed in dihedral group rings at every length a multiple of eight up to and including length forty-eight, and at some lengths beyond forty-eight. We also successfully investigate the possibility to construct some type I codes in the same way.

Structural and Universality Properties of Gelfand–Tsetlin Patterns and Their Generalizations

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This is an abstract of the PhD thesis *Structural and universality properties of Gelfand–Tsetlin patterns and their generalizations* written by Anthony Paul Metcalfe under the supervision of Prof. Neil

O'Connell at the Mathematics Department of University College Cork and submitted in March 2009.

This thesis is primarily concerned with probability distributions on sets of interlaced particles. Such distributions arise naturally throughout the study of random matrix theory, most famously when considering the eigenvalues of the principal minors of random Hermitian matrices.

We construct a random walk on sets of interlaced particles on the discrete circle, using a dynamical rule inspired by the *Robinson–Schensted–Knuth*, or *RSK*, correspondence. Using this, we prove analogous results on the circle to some recently discovered results on sets of interlaced particles on the line.

We construct a measure on the set of interlaced particle configurations on the infinite cylinder. We prove a determinantal structure, and compute an associated space-time correlation kernel. We consider models which are equivalent to the above construction, namely tilings of the cylinder with rhombi, configurations of non-intersecting paths, and dimer configurations of the honeycomb lattice on the infinite cylinder.

Finally, we consider universality problems on sets of interlaced particles, that arise from free probability. We consider the asymptotic behavior of particles in the bulk of the configurations, as the size of the configurations increase. We consider configurations both on a line, and on a cylinder. Under certain assumptions, we show that the particles behave locally like a determinantal random point field, with correlation kernel given by the *sine* kernel.

Solving Polynomials with Integer Matrics

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This is an abstract of the PhD thesis *Solving Polynomials with Integer Matrices* written by Raja Mukherji under the supervision of Professor Thomas J. Laffey at the School of Mathematical Sciences, University College Dublin and submitted in October 2009.

For an $n \times n$ complex matrix A, and a polynomial $f(x)$ with complex coefficients, methods to find $n \times n$ complex matrices X satisfying $f(X) = A$ have been discussed for some time. A quite complete (and lengthy) discussion can be found in Evard and Uhlig's paper [2]. A brief summary of the relevant sections are given in Chapter 2.

In Chapter 3, we look at the case where A is an $n \times n$ rational or integer matrix and $f(x)$ has rational or integer coefficients. We first look at the rational case, and reduce the problem to the case where the characteristic polynomial of A is a power of an irreducible polynomial. We present a new method for enumerating and finding rational matrices X satisfying $f(X) = A$. We then discuss an existing method first described by Drazin [1]. We present a new method for finding rational solutions in the case when the characteristic polynomial of A is irreducible and conditions such that these solutions have only integer entries. We give a bound on the denominator(s) occuring in rational solutions and discuss how the bound can be used to find exact rational solutions by numerical methods.

In Chapter 4, we look at the case where A is an $n \times n$ integer matrix and $f(x)$ has integer coefficients and show how the results of Chapter 3 can be applied to the integer case.

In Chapter 5, we look at the case where $f(x) = x^m$, and discuss solutions X satisfying $X^m = A$, where X has entries in an algebraic extension field of $\mathbb{Q}$. We present new methods to compute exact solutions in the cases where $m = 2$ and $n \leqslant 6$ and where $m = 3$ and $n \leqslant 4$. We discuss the minimum degree of such an extension field and show that for $m = 2$ and $n \leqslant 5$, there must exist such an extension field of degree a power of 2 over $\mathbb{Q}$. We show that in the case $m = 2$ and $n = 6$, there need not exist a extension field of degree a power of 2 over $\mathbb{Q}$ and present an example of such a matrix A.

The topic of matrix similarity arises in much of this discussion. Working over a field, in particular $\mathbb{C}$ or $\mathbb{Q}$, is straightforward, using well known results such as the Jordan and Frobenius (Rational) canonical forms. However, matrix similarity over a ring, even over $\mathbb{Z}$, is more complicated. For example, the Latimer–MacDuffee theorem [3, III.16], [4] relates similarity over $\mathbb{Z}$ to ideal classes in the ring of algebraic integers in a field extension. In chapter 6, we look at the idealizer of such ideals, and present new results about their relation to the centralizer of an integer matrix, and their role in similarity over $\mathbb{Z}$.

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Mathematics in the life of Éamon de Valera

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This is an abstract of the PhD thesis *Mathematics in the life of Éamon de Valera* written by Cáit Ní Shúilleabháin under the supervision of Professor Des MacHale and Mr. Gabriel Doherty at the School of Mathematical Sciences and Department of History at University College Cork and submitted in November 2009.

The objective of the thesis was to examine, in detail, the influence of mathematics in the life of Éamon de Valera. Throughout this abstract I will give a brief insight into some of the ways his interest in mathematics impacted on his private and public life. At the turn of the century, de Valera entered University College Blackrock with the intention of pursuing a degree in mathematics and mathematical physics. In his final year, he received an offer, which he accepted, of teaching in Blackrock's sister College in Rockwell, Co. Tipperary. It was reported that de Valera was a very talented teacher, and therefore, he was entrusted with teaching both Senior Grade students and undergraduate degree students, though he had to finish his own degree! The pressures of a full teaching load left him little time to study and as a result, he had to be content with a pass degree. This made further study difficult for him; however, he benefited from public lectures given by UCD lecturers Arthur Conway and Henry MacWeeney, and lectures given by the Astronomer Royal, E.T. Whittaker. After meeting Conway de Valera's interest in quaternions intensified. Conway reported that "he [de Valera] has in the past two years [1910–1912] gone deeply into the subject of Quaternions, and is at present prosecuting an important original research in them which promises to be of considerable interest." During February 1917 de Valera wrote a letter to Conway on the subject of celestial mechanics. The letter also contained a reference to the type of mathematics de Valera had been working on

with Conway, namely conformal representation. It is difficult to ascertain what type of problems he was working on, but it may have been that he was working on the application of conformal representation to quaternions. Arising from his 3rd level teaching experience, and his friendship with Conway and Whittaker, de Valera's confidence grew and he applied for the chair of Mathematical Physics in UCC in 1912. In the straw vote de Valera secured 11 votes, two higher than the nearest competitor, E. H. Harper. However, in the actual vote they both scored 10 and the two names were sent along with the results of the final vote. Harper was elected to the chair, and de Valera managed to secure a post teaching mathematical physics in the recognised College of the NUI in Maynooth. Due to de Valera's participation in the 1916 rising, his formal study of mathematics ceased. He did, however, maintain an interest in the subject throughout his life and in some scenario's actively used mathematics to solve political problems. One instance of this can be found when he set about devising external association, a concept which would satisfy both Irish and British aspirations following the 1921 War of Independence. Although not all accepted his proposal, certain republicans, though not enamoured with it, were eventually won over by de Valera's mathematical explanations, which were based on set theory. The most important contribution de Valera made to mathematics both in Ireland and internationally was the foundation of the Dublin Institute for Advanced Studies in 1940. Upon founding the Institute, de Valera hoped that Ireland would "achieve a reputation comparable to the reputation which Dublin and Ireland had in the middle of the last century", which, it appears was the case according to former director, JL Synge, who maintained that "former scholars are to be found in chairs in many parts of the world, and the school may feel reasonably proud that Ireland has, with profit to herself, made an international contribution to physics". The foundation of DIAS, among other acts, culminated in de Valera's election as an honorary Fellow of the Royal Society in 1968. Though de Valera may not have had the opportunity to study mathematics formally after his entry into politics, he often practiced it for fun. His attitude towards mathematics can be summed up by Rényi, who once said, "If I feel unhappy, I do mathematics to become happy. If I am happy, I do mathematics to keep happy."

The Hochschild Cohomology Ring of a Quadratic Monomial Algebra

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This is an abstract of the PhD thesis *The Hochschild Cohomology Ring of a Quadratic Monomial Algebra* written by David O'Keeffe under the supervision of Dr. Emil Sköldberg at the Department of Mathematics, National University of Ireland, Galway and submitted in January 2009.

Until recently, little was known about the multiplicative structure of the Hochschild cohomology ring for most associative algebras. During the last decade or so, more light has been shed on this topic, where several papers have been published regarding the structure of the Hochschild cohomology rings for various algebras.

In this thesis, we compute the Hochschild cohomology groups for a special class of associative algebras, the so-called class of Quadratic monomial algebras. These results are used to compute the Hochschild cohomology ring for the above class of algebras. We extend results obtained by Claude Cibils, where he computes the cohomology ring for the class of radical square zero algebras. The latter class of algebras form a subclass of the class of quadraric monomial algebras.

Chapter 1 consists of all the necessary background material that is referred to throughout this work. There are also some new proofs of already known results. However most of the original work is contained in chapters 2, 3 and 4.

Chapter 2 describes the Hochschild cohomology groups of a quadratic monomial algebra in terms of the cohomology of a graph. An alternative calculation of these groups was performed by Emil Sköldberg. Also in this chapter we describe the generating elements of the cohomology algebra.

Chapter 3 builds on the calculations of the previous chapter by computing the algebra structure on the cohomology algebra. We describe the product structure on the cohomology algebra as a composition of chain maps on a projective resolution.

Finally using the algebra structure calculated in Chapter 3 and the Hilbert series of a vector space, we show in Chapter 4, the cohomology algebra of a quadratic monomial algebra exhibits all possible behaviours as an algebra: It may be

- finite dimensional over k,
- finitely generated over k,
- infinitely generated over k,

where we write k to denote a field of arbitrary characteristic.

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Stochastic Delay Difference and Differential Equations: Applications to Financial Markets

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This is an abstract of the PhD thesis *Stochastic Delay Difference and Differential Equations: Applications to Financial Markets* written by Catherine Swords under the supervision of Dr John Appleby at the School of Mathematical Sciences, Dublin City University and submitted in September 2009.

This thesis deals with the asymptotic behaviour of stochastic difference and functional differential equations of Itô type. Numerical methods which both minimise error and preserve asymptotic features of the underlying continuous equation are studied. The equations have a form which makes them suitable to model financial markets in which agents use past prices. The second chapter deals with the behaviour of moving average models of price formation. We show that the asset returns are positively and exponentially correlated, while the presence of feedback traders causes either excess volatility or a market bubble or crash. These results are robust to the presence of nonlinearities in the traders demand functions. In Chapters 3 and 4, we show that these phenomena persist if trading takes place continuously by modelling the returns using linear and nonlinear stochastic functional differential equations (SFDEs). In the fifth chapter, we assume that some traders base their demand on the difference between current returns and the maximum return over

several trading periods, leading to an analysis of stochastic difference equations with maximum functionals. Once again it is shown that prices either fluctuate or undergo a bubble or crash. In common with the earlier chapters, the size of the largest fluctuations and the growth rate of the bubble or crash is determined. The last three chapters are devoted to the discretisation of the SFDE presented in Chapter 4. Chapter 6 highlights problems that standard numerical methods face in reproducing long-run features of the dynamics of the general continuous-time model, while showing these standard methods work in some cases. Chapter 7 develops an alternative method for discretising the solution of the continuous time equation, and shows that it preserves the desired long-run behaviour. Chapter 8 demonstrates that this alternative method converges to the solution of the continuous equation, given sufficient computational effort.

Homology Stability for the Special Linear Group of a Field and Milnor–Witt K-theory

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This is an abstract of the PhD thesis *Homology Stability for the Special Linear Group of a Field and Milnor–Witt K-theory* written by Liqun Tao under the supervision of Dr. Kevin Hutchinson at the School of Mathematical Sciences, UCD and submitted in February 2009.

The main results of the thesis provide answers, for fields of characteristic zero, to some old questions of A. A. Suslin and C.-H. Sah about homology stability for the special linear group and to a more recent question of J. Barge and F. Morel as to whether Milnor–Witt K-theory arises as the first obstruction to homological stability.

Let F be an infinite field and let $f_{t,n}$ be the stabilization homomorphism $\mathrm{H}_n(\mathrm{SL}_t(F), \mathbb{Z}) \to \mathrm{H}_n(\mathrm{SL}_{t+1}(F), \mathbb{Z})$. C.-H. Sah [1] conjectured that $f_{t,n}$ is an isomorphism for all $t \geq n+1$. For $n \geq 2$ even, Barge and Morel [2] defined a natural surjective map from the cokernel of $f_{n-1,n}$ to the n-th Milnor–Witt K-theory group $K_n^{\mathrm{MW}}(F)$, and asked whether this map is an isomorphism for all infinite fields.

In this thesis, the following results are proved: Let F be a field of characteristic 0. Then $f_{t,n}$ is an isomorphism for $t \geq n+1$ and is surjective for $t = n$. When n is odd, $f_{n,n}$ is an isomorphism. When

n is even, the kernel of $f_{n,n}$ is naturally isomorphic to $I^{n+1}(F)$, the $(n+1)$st power of the fundamental ideal of the Witt Ring of the field F. When $n \geq 2$ is even, the question of Barge and Morel has a positive answer. When $n \geq 3$ is odd, then the cokernel of $f_{n-1,n}$ is naturally isomorphic to the subgroup $2K_n^{\mathrm{M}}(F)$ in the (additive) Milnor K-theory group $K_n^{\mathrm{M}}(F)$.

The thesis also proves a simple presentation of the additive group $K_1^{\mathrm{MW}}(F)$, which is closely related to Hermitian K-theory.

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On the Pathwise Large Deviations of Stochastic Differential and Functional Differential Equations with Applications to Finance

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This is an abstract of the PhD thesis *On the pathwise large deviations of stochastic differential and functional differential equations with applications to finance* written by Huizhong Wu under the supervision of Dr John Appleby at the School of Mathematical Sciences, Dublin City University and submitted in September 2009.

The thesis deals with the asymptotic behaviour of highly nonlinear stochastic differential equations, as well as linear and nonlinear functional differential equations. Both ordinary functional and neutral equations are analysed. In the first chapter, a class of nonlinear stochastic differential equations which satisfy the Law of the Iterated Logarithm is studied, and the results applied to a financial market model. Mainly scalar equations are considered in the first chapter. The second chapter deals with a more general class of finite-dimensional nonlinear SDEs and SFDEs, employing comparison and time change methods, as well as martingale inequalities, to determine the almost sure rate of growth of the running maximum of functionals of the solution. The third chapter examines the

exact almost sure rate of growth of the large deviations for affine stochastic functional differential equations, and for equations with additive noise which are subject to relatively weak nonlinearities at infinity. The fourth chapter extends conventional conditions for existence and uniqueness of neutral functional differential equations to the stochastic case. The final chapter deals with large fluctuations of stochastic neutral functional differential equations.