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NCCA Review of Mathematics in Post-Primary Education

Response of the Royal Irish Academy Committee
for Mathematical Sciences

Question 1: Role and Purpose of Mathematics Education

Please comment on the level of importance you would attach to post-primary Mathematics Education and your views regarding its role and purpose.

Mathematics is vitally important to the entire population. The goal of Mathematics Education is to provide a good Mathematics experience for all students so as to

– contribute to the personal development of students;

– prepare students for life and work;

– prepare students for further/higher education.

It could be argued that any field of academic study contributes to a student's personal growth, however the abstract nature and rigour of Mathematics places it in a unique position for the development of thinking and problem solving skills. Mathematics is fundamental to our understanding of the world we live in, to explain it and to predict its future. Furthermore, Mathematics is essential to the advancement of Science, Technology, Commerce and Industry. It is good that, presently, Irish students study Mathematics throughout their school careers, and thus for a longer time than students in other industrialised countries.

Question 2: Concerns Regarding Mathematics

The discussion paper identifies concerns about: the emphasis on procedural skills rather than on understanding of Mathematics; the poor application of Mathematics in real-world contexts; the low uptake of Higher Level Mathematics, especially in Leaving Certificate; the low grades achieved at Ordinary Level, especially in Leaving Certificate; gender differences in uptake and achievement in Mathematics; difficulties in Mathematics experienced by some students in third-level courses. We would welcome your views on these issues or other concerns that you may wish to raise.

Some emphasis on procedural skills is important so that students may 'do' Mathematics successfully and effortlessly. However an over-emphasis on procedures in Irish classrooms is ultimately to the detriment of students' understanding. It is very important to stress the 'why' component of Mathematics. When looking at a piece of Mathematics the students need to ask 'Why is this useful?' and 'Why is this true?'. The first question is important so the students see the relevance of the subject, but the second question is equally important since otherwise Mathematics becomes a series of tricks. There is a real need to explain the ideas behind the procedures. If students really understood the Mathematics it would inevitably help with their grades, their uptake of Higher Level courses and their ability to apply Mathematics to real world problems. We should aspire to change the learning culture of students. This is best started at Primary Level, before the students learn not to look for meaning in what they do.

The Mathematics curriculum should be underpinned by a consistent vision of the subject that allows for an integrated approach from Primary through to Senior Cycle. The 'strands' term used at Primary Level is useful here. We would advocate that the content of the syllabi at all three levels is aligned in strands which run from Primary Level to Senior Cycle wherever possible (it seems reasonable that new strands would emerge later in the cycle). This 'strands' approach can deal effectively with content issues including Applied Mathematics topics, but needs further elaboration in terms of mathematical processes. The processes can be identified as cross-cutting the strands and should include skills, logical analysis, problem solving, modelling, optimising etc. This treatment also allows for a clear articulation of curriculum across important transition points, for example Primary Mathematics to Junior Cycle Mathematics, and Junior Cycle to Senior Cycle Mathematics.

Students' understanding of the relevance of Mathematics could be enhanced with an increased emphasis on applications. In particular students should be given opportunities to engage in mathematical modelling, as mathematical models are extensively used and students should appreciate this. A detailed syllabus specifying the application component should be devised and students should receive instruction and appropriate assessment in this area of the subject. The current lack of these components means, in particular, that students would experience difficulty in relating Mathematics to Science

and Technology, both at school and in later life.

The low uptake in Higher Leaving Certificate Mathematics can be explained in part by the fact that only 41% of students take Higher Junior Certificate Mathematics and 44% of them (or 18% of the total cohort) achieve a grade B or higher. Moreover, the Higher Leaving Certificate course is perceived as being difficult and is thought to impose a heavier workload than other Leaving Certificate subjects. Thus it is no surprise that only 17% of students take Higher Leaving Certificate Mathematics. We need to encourage more Junior Certificate students to take the Higher course. Low uptake in the Junior cycle stems in part from transition issues from Primary Mathematics and the quality of Mathematics teaching at this level, resulting in a poor mathematical experience for a lot of students.

The difficulties that students experience in Third Level are due to mathematical under-preparedness, in terms of mathematical knowledge and skills as well as attitudes. The Leaving Certificate Ordinary Level course is not working well for students in this regard and needs attention.

Question 3: Recent Developments in Mathematics Education in Ireland

In the past five years, a revised Mathematics curriculum has been implemented in primary schools and there has also been a syllabus revision in Junior Certificate Mathematics. Please comment on the impact of these changes and whether they go far enough to address the problems in Mathematics that have been identified.

There is a general agreement that the changes made at Primary and Junior Cycle levels have been positive, and that relating Mathematics to the real world can only help students' understanding of the material. It might, however, still be too early to assess the full impact of the changes. It would be desirable to have a coherent curriculum which spans both Primary and Second Level. The Primary and Junior Certificate curricula are accompanied by detailed teaching notes setting out examples of good practice, and this could be extended to the Senior Cycle.

Question 4: Current Trends in Mathematics Education

The discussion paper describes some of the approaches that are used in Mathematics Education, including 'modern Mathematics' with its emphasis on abstraction, logical structure, rigorous argument, set theory, number theory, etc and real-world or context-based Mathematics, also referred to as 'realistic Mathematics Education' or RME. Please comment on the relative merits of such approaches for Junior Certificate and Leaving Certificate Mathematics courses.

Abstraction, logical structure and rigorous argument are central to Mathematics, and are therefore essential components of a mathematical education. Students who cannot abstract or reason logically know little about Mathematics. However, to ignore applications is to fail to demonstrate to students the power and scope of the subject. A combination of 'Modern Maths' and RME would be the most desirable approach. It would be prudent to study the action taken in this regard in other countries, such as the Netherlands, Germany, the USA and Australia. It is important that students at Higher Junior Certificate should be exposed to proof. These students need to able to construct simple arguments themselves as opposed to just memorising proofs, so that they can truly develop thinking and problem solving skills.

Question 5: Mathematics in Relation to Other Subjects

The discussion paper notes the dual nature of Mathematics. It is geared to applications but is also worthy of study in its own right. It plays an important role in other subjects, especially the Science and Technology subjects. Please comment on this dual nature and on the relationship between Mathematics and other subjects, including the contribution that Mathematics can make to other subjects and their contributions to Mathematics.

Both aspects of the 'dual nature' of Mathematics are important and should be given due weight. However, a curricular mindset which sees Mathematics only as a service subject would be unfortunate. If Mathematics is to be applied in other subject areas it is necessary for students to understand the Mathematics rather than just to have a proficiency with procedural techniques. Dealing with new situations requires new procedures, and only a true understanding of Mathematics can lead to such new procedures being created.

When Mathematics is used in other subjects it is important that the same language and vocabulary be used wherever possible, so the students can see the connections between what they learn in Mathematics classes and topics they see elsewhere. Attention would need to be given to the syllabi of other subjects so as to allow coordination with Mathematics. Applying Mathematics across the curriculum requires specific attention over a long period of time by appropriately trained Mathematics teachers. This activity is not a by-product of curriculum change.

The vital role of Mathematics in the Physical Sciences and Technology is well-known, although the growing importance of the subject in the Biosciences and Finance does not appear to be well appreciated in secondary schools.

Question 6: Provision and Uptake of Mathematics

Internationally, the proportion of students who study Mathematics in senior cycle or upper second-level education is lower than in Ireland. In some countries, where a high proportion of students remain in school, students can choose between 'general' and 'specialist' Mathematics courses. In Ireland, practically all students in post-primary schools study Mathematics. However, the proportion of students taking the higher level syllabus is lower than had been expected when the three syllabus levels were introduced. Please comment on the adequacy of the current Mathematics courses in meeting the needs of all students in (i) *the Junior cycle and* (ii) *the Senior cycle of post-primary education. We would also welcome your views on the relatively low uptake in Higher Level Mathematics and any suggestions you might have for increasing this uptake.*

It is good that all Irish students study Mathematics to a later stage than in other countries, and this should continue. The three-level structure is also good as it allows for differentiation by ability, and the syllabi for all three levels are generally reasonable.

The Higher Level Leaving Certificate uptake is low, but this can be explained in part by Higher Junior Certificate results. It is important that students are not discouraged from taking Higher Junior Certificate at an early stage. Students' early exposure to Mathematics has a profound effect on their later choices. Continuing professional development for teachers could go some way towards providing a better classroom experience of Mathematics for such students.

There is also a particular concern over the proportion of Mathematics teachers whose degree does not contain a significant Mathematics component. Measures that could be taken to remedy the situation include giving priority to Mathematics graduates in Higher Diploma in Education programmes, providing inducements to Mathematics graduates to enter the teaching profession, and insisting that only persons qualified in the subject be allowed to teach Mathematics. The perception of Mathematics by the general public also has a role to play in influencing students' choices. A high-level programme to raise public awareness of Mathematics and its place in society should be undertaken immediately. There is also a need to provide better careers information for parents and students.

Question 7: Influence of the Examination Papers

Research has shown that teaching and learning in Mathematics is strongly influenced by the examination papers, with firm evidence of 'teaching to the test' (this is true for other subjects also). The absence of other forms of assessment, such as coursework, is noted as contributing to this dominant influence. Please give us your views on the assessment of Mathematics.

Currently public examinations are highly valued in Irish education and enjoy wide support in Irish society. However it is generally acknowledged that high-stakes examinations such as the Leaving Certificate and the points system are combining to have a negative impact on Second-Level education. It is well known that 'what you assess is what you get'. Students will only work at what bears immediate fruit and some parents and teachers encourage them in this practice. It is necessary to break out of this cycle.

The examination needs to be less predictable. At the moment it seems easy for teachers/students/media to predict the format of the paper and even the individual questions. The fact that 'question 1 is always about topic X' reinforces the notion that rote learning is the way to score highly. The rigidity of the exam also means that students can afford not to study certain topics, safe in the knowledge that they will still be able to answer other questions. The idea of a comprehension test should be explored.

The introduction of assessment of coursework would need very careful consideration. It has disadvantages as well as advantages, not least as regards who does the work. Detailed study of its imple-

mentation elsewhere would be needed in order to choose a workable model and one that teachers could support.

Question 8: Syllabus Levels and Range of Courses

The discussion paper points to some issues that have arisen in relation to the existence of three syllabus levels in Mathematics (Irish is the only other subject with a foundation level). Views vary greatly on issues such as nonrecognition of Foundation level mathematics for entry to many third-level courses, or the challenge of meeting demands for both 'general' and 'specialist' Mathematics courses in the one subject (the uptake of Leaving Certificate Applied Mathematics is very low). Please comment on these issues and on how they might be addressed within the current review.

As noted in 6 above, the three-level structure is good and should be maintained. However, in the interests of equity and the quality of the students' Mathematics experience, it seems reasonable to suggest that Foundation Mathematics should be recognised for entry to some specified courses at Third Level.

The Applied Mathematics course at Leaving Certificate needs urgent review. It is really a course on Theoretical Mechanics, which in turn is a branch of Mathematical Physics. This is one of many branches of Applied Mathematics. Since the course was devised the range of application of Mathematics has increased dramatically. The course could be made more accessible by developing topics for which the underlying Mathematics would be well within the compass of fifth-year students. In addition the appeal could be broadened beyond application in Physics to other areas of Science such as Biology and Chemistry, and to Engineering, Business and Finance, and the Humanities. Simple applications based on dynamic programming, difference equations and stochastic processes are quite accessible. Such changes in syllabus would, however, require substantial teacher education and support.

Question 9: Student Achievement in Mathematics

Section 5 of the discussion paper considers the results of the Junior Certificate and Leaving Certificate Mathematics examinations in recent years and the Chief Examiner's reports, as well as evidence from cross-national studies. It is noted that higher performing Irish students do less well than their counterparts in countries which record

comparable overall levels of achievement in international Mathematics tests. How effective, in your view, would each of the following measures be in improving the performance in Mathematics examinations?

(i) *Allocation of more class time to Mathematics.*

(ii) *Better pre-service and in-service education for teachers of Mathematics.*

(iii) *Improved Mathematics textbooks and other learning resources.*

(iv) *Provision of learning support for students who are experiencing difficulties with the subject.*

(v) *Provision of 'general' as well as 'specialist' Mathematics courses.*

(vi) *Increased emphasis in examination questions on the application of Mathematics to real-world problems.*

(vii) *The introduction of additional forms of assessment, such as coursework.*

(viii) *Improving the perception of Mathematics among parents and the general public.*

There is a general agreement that (i)–(v) and (viii) would be very effective in improving students' exam performance. There is some disagreement about points (vi) and (vii).

Question 10: Teaching and Learning in Mathematics

Research in Irish classrooms indicates that Mathematics is taught and learned in a traditional manner, mainly involving teacher exposition or demonstration of procedural skills and techniques for answering examination-type questions, followed by student practice of these techniques (in class or as homework) using similar questions. There appears to be little or no emphasis on students understanding the Mathematics involved, or on its application in different or unfamiliar contexts. Please comment on the strengths and weaknesses of this approach. We would also welcome your views on the degree to which syllabus change, assessment change, teacher professional development and support would contribute to bringing about changes in teaching and learning.

While traditional teaching is necessary to some degree, it should not be the only teaching strategy employed. For example the use of

technology in the classroom could play an important role in diversifying the methods used to teach Mathematics. Greater exposure to situations where students have to draw upon and develop their problem solving skills and strategies would also be welcomed. Any move away from traditional teaching will take many teachers outside their comfort zone, and it will be vital that appropriate support is provided for them.

Traditional teaching, and in particular teaching to the test would seem to be, at least in part, driven by the current textbooks. There is a need for a new range of textbooks which emphasise explanations, theory and applications. A resource text for teachers which gives background information as well as suggestions on how to present material is required.

As noted in 6 above, there are concerns about the mathematical background of many teachers. New teachers and teachers with weak backgrounds need extra support, possibly through mentoring initiatives. Universities need to give special thought to the education of future Mathematics teachers.

Question 11: Attitudes to and Beliefs about Mathematics

The discussion paper raises, on a number of occasions, issues surrounding the perceptions, attitudes and beliefs that exist in relation to Mathematics, such as: the view that Mathematics is a difficult subject; negative attitudes to Mathematics including a fear of the subject; the perception and advocacy of Mathematics, particularly Higher Level Mathematics, as an elite subject for only the best students; research findings that suggest a connection between teachers' views of Mathematics and their approach to teaching it. We would welcome your views on these or other issues associated with Mathematics.

There is a public perception that Mathematics is difficult and that it is acceptable not to be good at it. This attitude needs to be changed, possibly by a Government-funded publicity campaign. There is also a perception, even among teachers, that one is either good at Mathematics or not, and that mathematics skills cannot be developed. This is a myth and needs to be addressed. Undoubtedly, there is a link between teachers' attitudes to Mathematics and how it is taught. If the attitudes of students to the subject are to be changed, we must start with teachers, their own attitudes and

teaching styles. This calls for a major and sustained programme of in-service and pre-service education.

Question 12: Other Influences

The discussion paper draws attention to a range of other cross-cutting themes or issues that affect Mathematics education in schools: cultural issues related to the value of education in general and Mathematics education in particular; equality issues; the points system; recent developments in and availability of information and communications technology in schools. Please comment on any of these issues, or on other factors that impact on Mathematics Education in schools.

The current points system appears to encourage students to take Leaving Certificate Mathematics at Ordinary Level, as achieving a grade 'A1' at Ordinary Level is perceived in many quarters to be easier than achieving a 'C3' at Higher Level. This is an issue which might benefit from revisiting.

As noted in 10 above, the use of ICT in the classroom warrants careful consideration.

Conclusions:

- In the short term, the predictability of the Leaving Certificate examination needs to be addressed.
- The provision of professional development for teachers is crucial to enhancing students' experience of Mathematics in the classroom.
- Encouraging more students to take the Higher Level option at Junior Certificate is key to increasing Higher Level uptake at Leaving Certificate.
- Meaningful application of Mathematics is a vital facet of students' mathematical education and should be included in syllabi at all levels.
- The Applied Mathematics syllabus for Leaving Certificate needs urgent review.