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Research Announcement

TAYLOR-MONOMIAL EXPANSIONS OF HOLOMORPHIC FUNCTIONS ON FRÉCHET SPACES

Seán Dineen

Let $\lambda := \lambda(A)$ denote a Fréchet nuclear spaces with Köthe matrix A and let $\{E_n\}_n$ denote a sequence of Banach spaces. Let $E := \lambda(\{E_n\}_n) := \{(x_n)_n : x_n \in E_n \text{ and } (\|x_n\|)_n \in \lambda(A)\}$ and endow E with the topology generated by the semi-norms

$$\|(x_n)_n\|_k := \sum_{n=1}^{\infty} a_{n,k}\|x_n\|, \quad k = 1, 2, \ldots$$

E is a Fréchet space and $\{E_n\}_n$ is an unconditional Schauder decomposition of E. Examples of spaces which can be represented in this fashion, include all Banach spaces and all Fréchet nuclear (and some Fréchet-Schwartz) spaces with basis. Let $H(E)$ denote the space of all $\mathbf{C}$-valued holomorphic functions on E and for $m \in N^{(N)}$, $m = (m_1, \ldots, m_n, 0 \ldots)$ let

$$P_m(x) = \frac{1}{(2\pi i)^n} \int_{|\lambda_i|=1} \frac{f(\sum_{i=1}^n \lambda_i x_i)}{\lambda_1^{m_1+1} \ldots \lambda_n^{m_n+1}} \, d\lambda_1 \ldots d\lambda_n$$

We have

$$f = \sum_{m \in N^{(N)}} P_m \qquad (*)$$

in the τ_0, τ_w, τ_δ topologies on $H(E)$.

The expansion $(*)$ reduces to the Taylor series expansion in the case of a Banach space (i.e. if $E_1 = E$, $E_n = 0$, $n > 1$) and to the monomial expansion for Fréchet nuclear spaces with a basis (when $\dim(E_n) = 1$ all n).

If $(E_n)_n$ is an unconditional Schauder decomposition for the Fréchet space E then the topology of E is generated by semi-norms satisfying

$$\left\|\sum_{n=1}^{\infty} x_n\right\| = \sup_{|\lambda_n|\le 1}\left\|\sum_{n=1}^{\infty}\lambda_n x_n\right\| \qquad (**)$$

If $(\beta_n)_{n=1}^{\infty}$ is a sequence of real numbers, $\beta_n \geq 1$ all n, let

$$\left\|\sum_{n=1}^{\infty} x_n\right\|_{\beta,j} = \left\|\sum_{n=1}^{j} x_n + \sum_{n=j+1}^{\infty}\beta_n x_n\right\|.$$

If $m \in N^{(N)}$ we let $\mathcal{P}_e(^m E)$ denote the set of all $|m|$-homogeneous polynomials on E which are homogeneous in the even variables i.e. if $m = (m_1, m_2, \ldots, m_n, \ldots)$ then $P \in \mathcal{P}_e(^m E)$ if and only if

(i) $P(\lambda x) = \lambda^{|m|}P(x)$ for all $x \in E$, $\lambda \in \mathbf{C}$.

(ii) $P\left(\lambda x_{2i} + \sum_{n=1}^{\infty} x_n\right) = \lambda^{m_{2i}} P\left(\sum_{n=1}^{\infty} x_n\right)$ for all i, all $x \in E$ and all $\lambda \in \mathbf{C}$.

Our main technical tool is the following proposition.

Proposition. *Let $\{E_n\}_n$ denote an unconditional Schauder decomposition for the Fréchet space E, let F denote a Banach space and let T denote an F-valued linear function on $H(E)$ which is bounded on the locally bounded subsets of $H(E)$. Let $\beta_n \geq 1$ all n, $\beta_{2n-1} = 2$ all n and suppose $\sum_{n=1}^{\infty} x_n \in E$ implies $\sum_{n=1}^{\infty}\beta_n^p x_n \in E$ for all $p > 0$. Let $\|\cdot\|$ denote a continuous semi-norm satisfying (**) and suppose there exists $C > 0$ such that*

$$\|T(P)\| \leq C\|P\| \quad \text{for all } P \in \mathcal{P}_e(^m E) \text{ and all } m \in N^{(N)}$$

where $\|P\| = \sup\{|P(x)|;\ \|x\| \leq 1\}$. Then, for any $\delta > 1$, there exists $C_1 > 0$ and a positive integer j such that

$$\|T(P)\| \leq C_1\delta^{|m|}\|P\|_{\beta,j}$$

for all $P \in \mathcal{P}_e(^m E)$ and all $m \in N^{(N)}$ where

$$\|P\|_{\beta,j} = \sup\{|P(x)|;\ \|x\|_{\beta,j} \leq 1\}.$$

Theorem. *Let $\lambda(A)$ denote a Fréchet-nuclear space with DN and let $\{E_n\}_n$ denote a sequence of Banach spaces each of which admits an unconditional finite dimensional Schauder decomposition. Then $\tau_w = \tau_\delta$ on $\mathcal{H}\big(\lambda(\{E_n\}_n)\big)$.*

A Fréchet nuclear space has DN if and only if it is isomorphic to a subspace of s. The above theorem includes the known cases where E is a Banach space with an unconditional basis and the case where E is a Fréchet-nuclear space with basis and DN. It also includes Fréchet–Schwartz spaces which are not nuclear. By considering complemented subspaces, we find that $\tau_w = \tau_\delta$ on $\mathcal{H}(E)$, where E is any reflexive subspace, with the approximation property, of a Banach space with an unconditional finite dimensional Schauder decomposition. This includes spaces which do not have a finite dimensional Schauder decomposition.

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