more for second–hand copies. In 1691, it is recorded that a copy was bought for about 2 guineas. It should be borne in mind that a good husbandman could be hired for between £3 and £4 a year (information from the *Mathematical Gazette* for December 1948).

Many of the books in *Bibliotheca Chemico–Mathematica* now cost between 500 and 1000 times what they cost 70 years ago. This is particularly true of books whose authors have become famous (like Babbage or Boole) or books produced in small print runs. (A sum invested at 10% interest compounded annually would increase 1000 fold in about 72 years.) On the other hand, some text books have really declined in value over the years when inflation is taken into account. For example, the first edition of Todhunter's *Analytical Statics* was published by Macmillan in 1853 and sold at 10s 6d, which seems expensive for the time. The fifth edition of the book was still on sale in 1890 at the same price. Today, this book would probably cost no more than £15, so that it has not held its value over 140 years. Boole's *Laws of Thought* was available from Sotheran's for £1 15s and it was described as *very scarce*. Group theorists who know the impact made in the last century by Camille Jordan's *Traité des substitutions*, published in 1870, may like to know that this work was available from Sotheran's for £3 7s 6d and described as *very scarce*. I have not heard of an original copy for sale in recent times.

In conclusion, I would like to make a small advertisement. I am interested in buying books, papers or magazines of a scientific or mathematical nature, preferably pre–20th century. If you have any such items that you wish to dispose of, you may think of contacting me at the address below.

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POLYNOMIALS AND SERIES IN BANACH SPACES*

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Abstract: We show that homogeneous polynomials acting on Banach spaces preserve weakly unconditionally Cauchy (w.u.C.) series and unconditionally converging (u.c.) series. This fact allows to define the class of unconditionally converging polynomials as those taking w.u.C. series into u.c. series. It includes most of the classes of polynomials previously considered in the literature. Then we study several "polynomial properties" of Banach spaces, defined by relations of inclusion between classes of polynomials. In our main result we show that a Banach space E has the polynomial property (V) if and only if for all $k \in \mathbf{N}$ the space of homogeneous scalar polynomials $\mathcal{P}(^kE)$ is reflexive; hence, its dual space E^*, like the dual of Tsirelson's space, is reflexive and contains no copies of ℓ_p.

Throughout the paper, E and F will be real or complex Banach spaces, B_E the unit ball of E and E^* its dual space. We will write $\mathbf{K}$ for the scalar field, which will be always $\mathbf{R}$ or $\mathbf{C}$, the real or the complex field, and $\mathbf{N}$ for the natural numbers. Moreover, $\mathcal{P}(E,F)$ will stand for the space of all (continuous) polynomials from E into F. Any polynomial $P \in \mathcal{P}(E,F)$ can be written as a sum of homogeneous polynomials: $P = \sum_{k=0}^n P_k$, with $P_k \in \mathcal{P}(^kE,F)$, the space of all k-homogeneous polynomials from E into F.

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We will only give here the main results and sketches of some of the proofs. A complete exposition, including detailed proofs, can be found in [5].

1. Preservation of series by polynomials

Recall that a series $\sum_{i=1}^{\infty} x_i$ in a Banach space E is *weakly unconditionally Cauchy* (in short, *w.u.C.*) if for every $f \in E^*$ we have $\sum_{i=1}^{\infty} | f(x_i) | < \infty$; equivalently, if

$$\sup_{|\epsilon_i|\leq 1} \left\| \sum_{i=1}^{\infty} \epsilon_i x_i \right\| < \infty.$$

The series $\sum_{i=1}^{\infty} x_i$ is *unconditionally convergent* (in short, *u.c.*) if any subseries is norm-convergent; equivalently, if

$$\lim_{n\to\infty} \sup_{|\epsilon_i|\leq 1} \left\| \sum_{i=n}^{\infty} \epsilon_i x_i \right\| = 0.$$

The series $\sum_{i=1}^{\infty} x_i$ is *absolutely convergent* if $\sum_{i=1}^{\infty} \|x_i\| < \infty$.

Clearly, absolutely convergent series are u.c.; however, by the Dvoretsky-Rogers theorem [4], any infinite-dimensional Banach space contains an u.c. series which is not absolutely convergent. For example, if e_n denotes the unit vector basis of ℓ_2, then the series $\sum_{n=1}^{\infty} e_n/n$ is u.c., but not absolutely convergent.

Also, any u.c. series is w.u.C., and the prototype example of w.u.C. series which is not u.c. is given by $\sum_{i=1}^{\infty} e_i$, where $\{e_i\}$ denotes the unit vector basis of the space c_0. In fact, for any w.u.C. series $\sum_{i=1}^{\infty} x_i$ which is not u.c. there exist natural numbers $m_1 < n_1 < \cdots < m_k < n_k < \cdots$ such that the sequence of blocks

$$y_k := x_{m_k} + \cdots + x_{n_k}$$

is equivalent to the unit vector basis of c_0. (See [4]).

In this section we give first an estimation of the unconditional norm of the image of a finite sequence by a homogeneous polynomial, from which we derive the preservation of w.u.C. series and u.c. series under the action of polynomials. Then we define the class of unconditionally converging polynomials, and compare it with other classes of polynomials that have appeared in the literature.

Lemma 1. *Given $k \in \mathbb{N}$ there exists $C_k > 0$ such that for any $P \in \mathcal{P}({}^kE, F)$ and $x_1, ..., x_n \in E$ we have*

$$\sup_{|\epsilon_j|\leq 1} \left\| \sum_{j=1}^{n} \epsilon_j P x_j \right\| \leq C_k \sup_{|\nu_j|\leq 1} \left\| P\left(\sum_{j=1}^{n} \nu_j x_j \right) \right\|.$$

We can take $C_k = 1$ in the complex case, and $C_k = (2k)^k/k!$ in the real case.

The proof of the case in which E and F are complex spaces relies on the properties of the generalized Rademacher functions $s_n(t)$, $n \in \mathbb{N}$, introduced in [3], which are step functions on the interval $[0,1]$ verifying [3] for any choice of integers $i_1, \ldots, i_k$; $k \geq 2$,

$$\int_0^1 s_{i_1}(t)...s_{i_k}(t)dt = \begin{cases} 1 & \text{if } i_1 = \cdots = i_k; \\ 0 & \text{otherwise.} \end{cases}$$

In the case of real spaces, the proof is obtained using the complexifications of the spaces, and the polarization identities relating a homogeneous polynomial and its associated symmetric multilinear map.

Using Lemma 1, it is not difficult to prove the following

Theorem 2. *Any polynomial $P \in \mathcal{P}(E, F)$ takes w.u.C. (u.c.) series into w.u.C. (u.c.) series.*

This result suggests introducing the following class of polynomials.

Definition 3. A polynomial $P \in \mathcal{P}(E, F)$ is said to be *unconditionally converging* if it takes w.u.C. series into u.c. series.

We shall denote by $\mathcal{P}_{uc}({}^kE, F)$ the class of all k-homogeneous unconditionally converging polynomials from E to F.

Observe that in the case of E or F containing no copies of c_0, any w.u.C. series in that space is u.c. [4]; hence $\mathcal{P}({}^kE, F) = \mathcal{P}_{uc}({}^kE, F)$. Moreover, we can characterize unconditionally converging polynomials in terms of the action on sequences equivalent to the unit vector basis $\{e_n\}$ of c_0.

Lemma 4. *For any $P \in \mathcal{P}(^kE, F)$ which is not unconditionally converging there exists an isomorphism $i : c_0 \to E$ such that $\{(P \circ i)e_n\}$ is equivalent to $\{e_n\}$.*

The proof uses the Bessaga-Pełczyński principle in order to select a basic sequence from certain blocks of a suitable w.u.C. series $\sum_{i=1}^{\infty} x_i$ such that $\sum_{i=1}^{\infty} Px_i$ is not u.c., and then applies Lemma 1.

Next we describe the relation between the class $\mathcal{P}_{uc}$ and other classes of polynomials considered in the literature.

Recall that $P \in \mathcal{P}(^kE, F)$ is *weakly compact*, denoted by $P \in \mathcal{P}_{wco}(^kE, F)$, if it takes bounded subsets into relatively weakly compact subsets, and P is *completely continuous*, denoted by $P \in \mathcal{P}_{cc}(^kE, F)$, if it takes weakly Cauchy sequences into norm convergent sequences. These classes were considered in [10] and [11].

Moreover, we shall consider the class $\mathcal{P}_{cc0}(^kE, F)$ of *completely continuous at 0* polynomials, formed by those $P \in \mathcal{P}(^kE, F)$ taking weakly null sequences into norm null sequences. Clearly $\mathcal{P}_{cc}(^kE, F) \subset \mathcal{P}_{cc0}(^kE, F)$, but in general (see Proposition 14) the containment is strict for $k > 1$ and E failing the Schur property.

Recall that $A \subset E$ is said to be a *Rosenthal set* if any sequence $(x_n) \subset A$ has a weakly Cauchy subsequence. In contrast with the case of linear operators, a polynomial taking Rosenthal sets into relatively compact subsets need not take weakly null sequences into norm null sequences, as it is shown by the scalar polynomial

$$P : (x_n) \in \ell_2 \longrightarrow \sum_{n=1}^{\infty} x_n^2 \in \mathbf{R}.$$

The converse implication also fails, since for the polynomial

$$Q : (x_n) \in \ell_2 \longrightarrow \left(\sum_{k=1}^{\infty} \frac{x_k}{k}\right)(x_n) \in \ell_2$$

we have that $Q(e_1 + e_n) = (1 + 1/n)(e_1 + e_n)$ has no convergent subsequences, although Q takes weakly null sequences into norm null sequences, because of the factor $(\sum_{k=1}^{\infty} x_k/k)$.

Finally, recall that $A \subset E$ is said to be a *Dunford-Pettis set* [2] if for any weakly null sequence $(f_n) \subset E^*$ we have $\lim_n \sup_{x \in A} |f_n(x)| = 0$. Using this class of subsets, we will say as in [5] that a polynomial $P \in \mathcal{P}(^kE, F)$ belongs to $\mathcal{P}_{wd}$ if and only if its restriction to any Dunford-Pettis subset of E, endowed with the inherited weak topology, is continuous.

Proposition 5. *A polynomial $P \in \mathcal{P}(^kE, F)$ belongs to $\mathcal{P}_{uc}$ in the following cases:*

(a) $P \in \mathcal{P}_{cc0}$.
(b) *P takes Rosenthal subsets of E into relatively compact subsets of F.*
(c) $P \in \mathcal{P}_{wd}$.
(d) $P \in \mathcal{P}_{wco}$.

The result in the cases (a) and (b) is an immediate consequence of Lemma 4, since the unit vector basis of c_0 is a weakly null sequence which forms a non relatively compact set.

Case (c) follows from Lemma 4 also, since given $P \in \mathcal{P}(^kE, F) \backslash \mathcal{P}_{uc}$, and an isomorphism $i : c_0 \to E$ such that $P \circ i \notin \mathcal{P}_{uc}(^kc_0, F)$, we have that $\{ie_n\}$ is a Dunford-Pettis set of E on which P is not weakly continuous; hence $P \notin \mathcal{P}_{wd}$.

Finally, we have $\mathcal{P}_{wco}(^kE, F) \subseteq \mathcal{P}_{wd}(^kE, F)$ (see [5]); hence (d) follows from (c).

2. Polynomial properties of Banach spaces

Pełczyński [10] introduced Banach spaces with the *polynomial Dunford-Pettis property* as the spaces E such that (with our notation) $\mathcal{P}_{wco}(^kE, F) \subseteq \mathcal{P}_{cc}(^kE, F)$ for any $k \in \mathbf{N}$ and F, and raised the question whether or not the polynomial Dunford-Pettis property coincides with the usual *Dunford-Pettis property*, which admits the same definition in terms of linear operators ($k = 1$). Ryan gave an affirmative answer in [11]. Moreover, Pełczyński [9] introduced Banach spaces with *property (V)* as the spaces E such that unconditionally converging operators from E into any Banach space are weakly compact.

In this section, by means of the class $\mathcal{P}_{uc}$ of unconditionally converging polynomials, we introduce and study the polynomial property (V) and other polynomial versions of properties of Banach spaces viz: the Dieudonné property, the Schur property, and property (V^*). We show that in contrast with the case of the Dunford-Pettis property, property (V) is very different from the polynomial property (V), since the prototype of space with this property is Tsirelson's space T^*. For the other polynomial properties, we show that sometimes the polynomial and the linear properties coincide, and sometimes not, with a general tendency of the polynomial property to imply the absence of copies of ℓ_1 in the space. Moreover, we obtain additional results relating $\mathcal{P}_{uc}$ and other classes of polynomials.

Definition 6. A Banach space E has the *polynomial property (V)* if for every k and F we have $\mathcal{P}_{uc}({}^kE,F) \subseteq \mathcal{P}_{wco}({}^kE,F)$.

It was shown in [9] that $C(K)$ spaces enjoy property (V). The next Lemma shows that this is not the case for the polynomial property.

Lemma 7. *If $\mathcal{P}_{uc}({}^kE,E) \subseteq \mathcal{P}_{wco}({}^kE,E)$ for some $k>1$, then E contains no copies of c_0.*

It has been shown that a Banach space E such that $\mathcal{P}({}^kE,\mathbf{K}) \equiv \mathcal{P}({}^kE)$ is reflexive for every $k \in \mathbf{N}$ has many of the properties of Tsirelson's space T^* [14]. In fact, E must be reflexive, and the dual space E^* cannot contain copies of ℓ_p ($1<p<\infty$). Note also that $\mathcal{P}({}^kT^*)$ is reflexive for every $k \in \mathbf{N}$ [1]. Next we present a characterization of the spaces E such that $\mathcal{P}({}^kE)$ is reflexive for some $k>1$ in terms of the class $\mathcal{P}_{uc}$ of polynomials.

Given $P \in \mathcal{P}({}^kE,F)$, we consider the associated conjugate operator defined by

$$P^* : f \in F^* \longrightarrow f \circ P \in \mathcal{P}({}^kE).$$

Moreover, we need the fact that for every Banach space E, the space $\Delta_\pi^k E$, defined as the closed span of $\{x \otimes \cdots \otimes x : x \in E\}$ in the projective tensor product $\hat{\otimes}_\pi^k E$, is a predual of the space of scalar polynomials $\mathcal{P}({}^kE)$ [12].

Theorem 8. *Given $k>1$, we have that the space $\mathcal{P}({}^kE)$ is reflexive if and only if $\mathcal{P}_{uc}({}^kE,F) \subseteq \mathcal{P}_{wco}({}^kE,F)$ for any F. In particular, E has the polynomial property (V) if and only if $\mathcal{P}({}^kE)$ is reflexive for every $k \in \mathbf{N}$.*

For the direct result it is enough to note that $P \in \mathcal{P}_{wco}$ if and only if the operator P^* is weakly compact [13].

For the converse, we derive from Lemma 7 that E contains no copies of c_0; hence $\mathcal{P}({}^kE,F) = \mathcal{P}_{uc}({}^kE,F) = \mathcal{P}_{wco}({}^kE,F)$ for any k and F, and then we observe that there exists a natural isomorphism between the space of polynomials $\mathcal{P}({}^kE,F)$ and the space of operators $L(\Delta_\pi^k E,F)$ which takes the weakly compact polynomials onto the weakly compact operators [12].

Extending the definition for operators, we shall say that $P \in \mathcal{P}({}^kE,F)$ is *weakly completely continuous*, denoted by $P \in \mathcal{P}_{wcc}({}^kE,F)$, if it takes weakly Cauchy sequences into weakly convergent sequences.

A Banach space E has the *Dieudonné property* if weakly completely continuous operators from E into any Banach space are weakly compact. Grothendieck [7] introduced this property and proved that $C(K)$ spaces enjoy it. The next result shows that, in general, $C(K)$ spaces fail to satisfy the polynomial Dieudonné property.

Proposition 9. *The following properties are equivalent:*

(a) E contains no copies of ℓ_1.
(b) $\mathcal{P}_{wcc}({}^kE,F) \subseteq \mathcal{P}_{wco}({}^kE,F)$ for any k and F.
(c) $\mathcal{P}_{cc}({}^kE,F) \subseteq \mathcal{P}_{wco}({}^kE,F)$ for any k and F.
(d) $\mathcal{P}_{cc}({}^kE,F) \subseteq \mathcal{P}_{wco}({}^kE,F)$ for some nonreflexive F and some $k>1$.

Corollary 10. *$\mathcal{P}_{wcc}({}^kE,F) \subseteq \mathcal{P}_{uc}({}^kE,F)$ for any $k \in \mathbf{N}$.*

Remark 11. It follows from Proposition 9 that, for any $k>1$, there is a polynomial $P \in \mathcal{P}_{cc}({}^k\ell_\infty,c_0)$ which is not weakly compact.

However, any operator from ℓ_∞ into c_0 is weakly compact and thereby completely continuous, since ℓ_∞ has the Dunford-Pettis property.

Then the question arises whether every polynomial from ℓ_∞ into c_0 is completely continuous.

As a complement of Theorem 8 we have the following

Theorem 12. *Given $k > 1$, we have $\mathcal{P}_{cc0}(^kE, F) \subseteq \mathcal{P}_{wco}(^kE, F)$ for any F if and only if $\mathcal{P}(^{k-1}E)$ is reflexive.*

Remark 13. In order to compare Theorems 8 and 12, we observe that for the sequence spaces ℓ_p the space of polynomials $\mathcal{P}(^k\ell_p)$ is reflexive if and only if $k < p < \infty$.

In fact, it was proved in [8] that for $k < p$, all polynomials in $\mathcal{P}(^k\ell_p)$ are completely continuous; hence, using a result of [12] (see [1]), we conclude that $\mathcal{P}(^k\ell_p)$ is reflexive. For $1 < p \leq k$ it is not difficult to show that $\mathcal{P}(^k\ell_p)$ contains a copy of ℓ_∞.

Recall that a Banach space E has the *Schur property* if weakly convergent sequences in E are norm convergent; equivalently, weakly Cauchy sequences are norm convergent. It is an immediate consequence of the definition that E has the Schur property if and only if $\mathcal{P}(^kE, F) = \mathcal{P}_{cc}(^kE, F)$ for any k and F. Next we give some other polynomial characterizations of Schur property.

Proposition 14. *The following properties are equivalent:*
(a) E has the Schur property.
(b) $\mathcal{P}_{uc}(^kE, F) \subseteq \mathcal{P}_{cc}(^kE, F)$ for any k and F.
(b') $\mathcal{P}_{cc0}(^kE, F) \subseteq \mathcal{P}_{cc}(^kE, F)$ for any k and F.
(c) $\mathcal{P}_{uc}(^kE, E) \subseteq \mathcal{P}_{cc}(^kE, E)$ for some $k > 1$.
(c') $\mathcal{P}_{cc0}(^kE, E) \subseteq \mathcal{P}_{cc}(^kE, E)$ for some $k > 1$.

Another property defined in terms of series is property (V^*), introduced in [10]. Recall that a subset $A \subset E$ is said to be a *(V^*) set* if for every w.u.C. series $\sum_{n=1}^\infty f_n$ in E^* we have

$$\lim_n \sup_{x \in A} |f_n(x)| = 0.$$

A Banach space E has *property* (V^*) if every (V^*) set in E is relatively weakly compact; equivalently, if any operator $T \in L(F, E)$, with unconditionally converging conjugate T^* is weakly compact.

Finally, we shall show that the polynomial version of the last formulation coincides with property (V^*). We shall denote by $\mathcal{P}_{uc*}(^kF, E)$ the class of all polynomials $P \in \mathcal{P}(^kF, E)$ such that P^* is unconditionally converging.

Proposition 15. *The following properties are equivalent:*
(a) E has property (V^*).
(b) For any k and any F, we have $\mathcal{P}_{uc*}(^kF, E) \subseteq \mathcal{P}_{wco}(^kF, E)$.
(c) For some k, we have $\mathcal{P}_{uc*}(^k\ell_1, E) \subseteq \mathcal{P}_{wco}(^k\ell_1, E)$.

In the proof we need the fact that given $P \in \mathcal{P}(^kF, E)$, the conjugate P^* is unconditionally converging if and only if $P(B_F)$ is a (V^*) set.

Remark 16. Part of the above results can be extended to holomorphic maps on Banach spaces. For example, holomorphic maps preserve u.c. series and w.u.C. series fulfilling natural restrictions, and if we define the *holomorphic property (V)* in the natural way, we can prove that it coincides with the polynomial property (V). For the details we refer to [5].

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References

[1] R. Alencar, R. M. Aron and S. Dineen, *A reflexive space of holomorphic functions in infinitely many variables*, Proc. Amer. Math. Soc. **90** (1984), 407–411.

[2] K. T. Andrews, *Dunford-Pettis sets in the space of Bochner integrable functions*, Math. Ann. **241** (1979), 35–41.

[3] R. M. Aron and J. Globevnik, *Analytic functions on c_0*, Revista Matemtica Univ. Complutense Madrid **2** (1989), 27–33.

[4] J. Diestel, Sequences and series in Banach spaces, (Graduate Texts in Mathematics 92). Springer-Verlag, 1984.

[5] M. González and J. M. Gutiérrez, *Weakly continuous mappings on Banach spaces with the Dunford-Pettis property*, J. Math. Anal. Appl. (to appear).

[6] M. González and J. M. Gutiérrez, *Unconditionally converging polynomials on Banach spaces* (to appear).

[7] A. Grothendieck, *Sur les applications linéaires faiblement compactes d'espaces du type $C(K)$*, Canad. Math. J. **5** (1953), 129–173.

[8] A. Pełczyński, *A property of multilinear operations*, Studia Math **16** (1957), 173–182.

[9] A. Pełczyński, *Banach spaces on which every unconditionally converging operator is weakly compact*, Bull. Acad. Polon. Sci. **10** (1962), 641–648.

[10] A. Pełczyński, *On weakly compact polynomial operators on B-spaces with Dunford-Pettis property*, Bull. Acad. Polon. Sci. **11** (1963), 371–378.

[11] R. A. Ryan, *Dunford-Pettis properties*, Bull. Acad. Polon. Sci. **27** (1979), 373–379.

[12] R. A. Ryan, Applications of topological tensor products to infinite dimensional holomorphy, (Ph. D. Thesis, Trinity College, Dublin, 1980), 1980.

[13] R. A. Ryan, *Weakly compact holomorphic mappings on Banach spaces*, Pacific J. Math. **131** (1988), 179–190.

[14] B. S. Tsirelson, *Not every Banach space contains an embedding of ℓ_p or c_0*, Functional Anal. Appl. **8** (1974), 138–141.

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Research Announcement

TAYLOR-MONOMIAL EXPANSIONS OF HOLOMORPHIC FUNCTIONS ON FRÉCHET SPACES

Seán Dineen

Let $\lambda := \lambda(A)$ denote a Fréchet nuclear spaces with Köthe matrix A and let $\{E_n\}_n$ denote a sequence of Banach spaces. Let $E := \lambda(\{E_n\}_n) := \{(x_n)_n : x_n \in E_n \text{ and } (\|x_n\|)_n \in \lambda(A)\}$ and endow E with the topology generated by the semi-norms

$$\|(x_n)_n\|_k := \sum_{n=1}^{\infty} a_{n,k}\|x_n\|, \quad k = 1, 2, \ldots$$

E is a Fréchet space and $\{E_n\}_n$ is an unconditional Schauder decomposition of E. Examples of spaces which can be represented in this fashion, include all Banach spaces and all Fréchet nuclear (and some Fréchet-Schwartz) spaces with basis. Let $H(E)$ denote the space of all **C**-valued holomorphic functions on E and for $m \in N^{(N)}$, $m = (m_1, \ldots, m_n, 0\ldots)$ let

$$P_m(x) = \frac{1}{(2\pi i)^n} \int_{|\lambda_i|=1} \frac{f(\sum_{i=1}^n \lambda_i x_i)}{\lambda_1^{m_1+1} \ldots \lambda_n^{m_n+1}} \, d\lambda_1 \ldots d\lambda_n$$

We have

$$f = \sum_{m \in N^{(N)}} P_m \qquad (*)$$

in the τ_0, τ_w, τ_δ topologies on $H(E)$.

The expansion $(*)$ reduces to the Taylor series expansion in the case of a Banach space (i.e. if $E_1 = E$, $E_n = 0$, $n > 1$) and to the monomial expansion for Fréchet nuclear spaces with a basis (when $\dim(E_n) = 1$ all n).