CONFERENCES AT UNIVERSITY COLLEGE DUBLIN
September 1994

7th Annual Meeting of the Irish Mathematical Society

5–6 September 1994

Speakers: J. M. Anderson (London), P. M. Gauthier (Montreal), B. Goldsmith (DIT), A. J. O'Farrell (Maynooth), J. V. Pulé (UCD), R. Ryan (UCG).

Requests for accommodation should be submitted by 1 July, 1994.
Conference dinner on Monday 5 September, 1994.
Further information: S. Dineen, S. Gardiner (addresses below).

Polynomials and Holomorphic Functions on Infinite Dimensional Spaces

7–9 September, 1994

Further information: S. Dineen, P. Mellon, C. Boyd.

Tel:	+353 1 706 8242
	+353 1 706 8265
Fax:	+353 1 706 1196
email:	sdineen@irlearn.bitnet
	gardiner@irlearn.bitnet

TRACE-ZERO MATRICES AND POLYNOMIAL COMMUTATORS

T. J. Laffey and T. T. West

Let $\mathbf{F}$ denote a field and $M_n(\mathbf{F})$ the algebra of $n \times n$ matrices over the field $\mathbf{F}$. If $X \in M_n(\mathbf{F})$, $\operatorname{tr}(X)$ will denote the trace of the matrix X. A well known result of Albert and Muckenhoupt [1] states that if $\operatorname{tr}(X) = 0$ then there exist matrices A, $B \in M_n(\mathbf{F})$ such that X is the commutator of A and B,

$$X = [A, B] = AB - BA.$$

Let p denote a polynomial in $\mathbf{F}[x]$ of degree greater than or equal to one. The *Polynomial Commutator* of A and B relative to p is defined to be

$$p[A, B] = p(AB) - p(BA).$$

It is easy to check, by examining the eigenvalues, that $\operatorname{tr}(p[A, B])$ is always zero. The Albert-Muckenhoupt result states that if $X \in M_n(\mathbf{F})$ with $\operatorname{tr}(X) = 0$ then, for $p(x) = x$,

$$X = p[A, B],$$

for some $A, B \in M_n(\mathbf{F})$. We show that, if the field $\mathbf{F}$ has characteristic zero the Albert-Muckenhoupt result may be extended to general polynomials of degree greater than, or equal to, one.

Theorem. *Let $\mathbf{F}$ be a field of characteristic zero and let $p \in \mathbf{F}[x]$ have degree greater than or equal to one. If $X \in M_n(\mathbf{F})$ is of trace zero then there exist matrices A, $B \in M_n(\mathbf{F})$ such that*

$$X = p[A, B].$$

First we prove the following elementary