

plane. A proof of the Seifert-Van Kampen theorem for polyhedra is given at the end of the chapter. Continuing the search for effective means of computing homology groups, Chapter 8 introduces CW complexes and their cellular homology. Chapter 9 begins with a statement (without proof) of the axiomatic characterization of homology theories due to Eilenberg and Steenrod, and then introduces enough homological algebra to prove the Eilenberg-Zilber theorem and Künneth formula for the homology of a product of spaces. Chapter 10 deals with covering spaces. The higher homotopy groups are studied in Chapter 11 using the suspension and loop functors. Results obtained include the exact homotopy sequence of a fibration, and its application to the fibration $S^3 \rightarrow S^2$ to show that the group $\pi_3(S^2)$ is non-trivial. The isomorphism $\pi_3(S^2) = \mathbf{Z}$ is beyond the scope of the book. In the final chapter a short discussion on de Rahm cohomology is used to motivate the study of the cohomology ring of a space.

The book is nicely structured, with explanations of where the theory is heading given at frequent intervals. Important definitions are often accompanied by a discussion on their origins. Many exercises are given at the end of sections. Proofs are usually given in full detail. Even though probably every result in the book (and many more besides) can be found in E.H. Spanier's classic text *Algebraic Topology*, J.J. Rotman's style of exposition makes the book a useful reference. However a lecture course based on this book may turn out to be a bit slow and dry. (Unfortunately the book corresponds to the syllabus of a one year course given at the University of Illinois, Urbana.) For example the homology of a space is defined on p. 66 but we have to wait until p. 157 until the homology of the torus is calculated, and until p. 226 for the homology of a lens space. The fundamental group is introduced on p. 44 but isn't calculated for a wedge of two circles until p. 171. Maybe too much rigour and generality in a first course on any topic is not a good thing!

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Book Review

INTRODUCTORY MATHEMATICS THROUGH SCIENCE APPLICATIONS

J. Berry, A. Norcliffe & S. Humble

Cambridge University Press, 1989,
stg£45 (hardback) ISBN 0 521 24119 7,
stg£15 (paperback) ISBN 0 521 28446 5.

Reviewed by Martin Stynes

For most of this century, pure (as opposed to applied) mathematics has held the centre of the mathematics stage. The last twenty years have seen a significant change in emphasis; today, applied mathematics is at least an equal partner. This trend has been reflected at the teaching level by the introduction of "new" topics such as discrete mathematics and dynamical systems, but it has not yet had much effect on the teaching of traditional courses such as calculus and linear algebra (except that sometimes these traditional courses disappear to make room for new courses). Textbooks for traditional courses now tend to use more applied material than heretofore, but the ratio of "applied" to "non-applied" examples is still low in the vast majority of cases. In this respect the book by Berry, Norcliffe & Humble is to be welcomed. Most of its examples are applied; they come from biology, chemistry and especially physics. As the authors state: "There is a growing awareness that we must not teach mathematics in isolation from its applications".

The book is intended for first-year service courses in science or engineering. It devotes approximately 150p. to pre-calculus material, 80p. to differentiation, 70p. to integration, 60p. to ordinary differential equations, 60p. to partial differentiation, and 100p. to



probability and statistics. The topics and techniques covered are quite standard.

Each chapter begins with a section entitled "scientific context", which seeks by example to motivate the material in that chapter. This motivation is an excellent idea, and overall it works well, but in some cases it becomes so involved as to discourage the learner. For instance, in Chapter 13 (Second-order ordinary differential equations), a cooling fin on a motor-cycle engine is modelled. This is quite an interesting example, but the explanation assumes a model for convection and Fourier's law for conductive heat flow, in order to derive a second-order ordinary differential equation.

The order of the material in the book is sometimes surprising. For example, on pp. 37-41, we meet the function e^x and learn that

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Then on p.44, the book explains why $a^m/a^n = a^{m-n}$ for $a > 0$ and m, n positive integers!

Returning to pp. 37-41, the authors are guilty here of presenting too many things at once. While learning about e^x , the novice reader encounters for the first time the binomial expansion, the limit of a sequence, the sigma notation, and the sum of an infinite series. It's all too much! Surely it would be better to discuss these other ideas before analyzing e^x ?

This reflects my main criticism of the text: its explanations are often not as clear as they could be. Sometimes they are misleading, as on p. 139, where the idea of $\lim_{x \rightarrow a} f(x)$ is being introduced: "...the value of $f = (x^2 - 1)/(x - 1)$ is not so obvious when $a = 1$... dividing top and bottom by $x - 1$ gives $f(x) = x + 1$. Now setting $x = 1$ we have $f(1) = 2$."

The discussion of points of inflection on pp. 194-6 is puzzling insofar as only points where the first derivative vanishes seem to be considered. This suspicion is confirmed on p. 467 where we read: "At a point of inflexion we know that both the first and second derivatives are zero". In fact on p. 467, it happens that at



the point in question (the critical point where the liquid, vapour and gas phases meet on the surface $p = f(v, t)$ given by van der Waal's equation of state) one has both derivatives vanishing, so the example is not in error; the harm is that the innocent reader will carry away a nonstandard definition of a point of inflexion.

While the book in its preface says that it does not claim to give a rigorous treatment, arguments are presented later as apparent proofs without any disclaimer. Thus the chain rule is justified by

$$\begin{aligned} (g(f(x)))' &= \lim_{h \rightarrow 0} \frac{g(f(x+h)) - g(f(x))}{h} \\ &= \lim_{h \rightarrow 0} \frac{g(f(x+h)) - g(f(x))}{f(x+h) - f(x)} \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= g'(f(x)) \cdot f'(x). \end{aligned}$$

I do think that this calculation has definite heuristic value, but the reader deserves a little warning!

To summarize, let me divide the book into examples, exercises and exposition. The material of the examples and exercises is very good, with many applications that were unfamiliar to me; a real effort is made to show how mathematics is used to solve problems in science and engineering. However, the exposition is only fair. The book is thus a useful source for lecture and examination material, but I would be reluctant to use it as a textbook.

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