Abstract of Doctoral Thesis

SYMMETRIC BANACH MANIFOLDS

Pauline Mellon

Banach manifolds are manifolds modelled locally on open subsets of complex Banach spaces. Symmetric Banach manifolds , or simply symmetric manifolds, are Banach manifolds with a norm on the tangent bundle and which have a high degree of symmetric structure. Namely, for every point of the manifold there is an involutive automorphism of the manifold which acts as a symmetry about that point. This structure is rich enough to allow a Riemann mapping type classification of the symmetric manifolds.

In finite dimensions the symmetric manifolds are exactly the Hermitian symmetric spaces. The Hermitian symmetric spaces were classified by Cartan in the 1930s , using Lie algebraic techniques and later by Koecher, Loos and others using Jordan algebraic techniques. There is a natural duality between the Hermitian symmetric spaces of compact and non-compact type. An analogue of this phenomenon also holds in infinite dimensions, even though the symmetric manifolds are then non-compact, as they are modelled on infinite dimensional Banach spaces.

Kaup gave an algebraic classification of the symmetric manifolds in the general case, by associating to each symmetric manifold a Banach space with an algebraic triple product, called a J^*-triple system or J^*-triple. He proved that the category of all simply-connected symmetric manifolds with base point is equivalent to the category of J^*-triple systems. These J^*-triples include in particular, all C^*-algebras, all JB^*- algebras and all J^*-algebras.

J^*-algebras are algebras of operators between Hilbert spaces which were introduced and studied by Harris. They give us a concrete setting in which to study J^*-triple phenomenon. The

techniques used by Harris are more function theoretic in nature, using spectral theory, the functional calculus *etc.*.

In this thesis we study two topics, both related to symmetric manifolds, one of which comes from the operator theoretic side of the subject and the other from the more abstract Jordan algebraic/Lie algebraic side.

Our aim in the first topic was to generalise results from J^*-algebras to J^*-triple systems, by replacing operator-theoretic techniques with Jordan algebraic techniques, while in the second, we adopted the opposite approach, by using some of the classical examples from operator theory to improve our understanding and to obtain results for the class of dual symmetric manifolds.

Our first topic deals with Schwarz-type inequalities for holomorphic mappings which were obtained by Ando, Fan and Włodarczyk in a series of papers culminating in various Julia-type lemmata and Wolff-type theorems for operator valued holomorphic mappings on J^*-algebras. Using Bergmann operators we obtained similar results for JB^*-triple systems. In realising these results for J^*-algebras in terms of the Jordan rather than the operator theoretic structure we appear to place the results in a more natural setting (even though the transition is not always smooth).

Our second topic is dual symmetric manifolds. These manifolds are non-compact, as they are modelled on infinite dimensional Banach spaces, but should intuitively behave like compact manifolds. To investigate this phenomenon we found it necessary to restrict ourselves to a certain class of JB^*-triple systems including in particular all commutative C^*-algebras of the form $C(X)$, for X a compact Hausdorff space.

If X is a compact Hausdorff space and U a JB^*-triple system then $C(X,U)$ is again a JB^*-triple system with pointwise defined triple product. If M with base point m_0 is the dual symmetric manifold of U then we show that the dual symmetric manifold of the JB^*-triple $C(X,U)$ is given by the universal covering manifold of

$$F_X(M) := \{ f \in C(X,M) : f \text{ is homotopic to the constant mapping } m_0 \text{ in } C(X,M) \}.$$

In particular for the commutative C^*-algebra $C(X)$ the dual symmetric manifold is the universal covering manifold of $F_X(\overline{C})$. We then show that the dual symmetric manifold of $C(X)$ displays the compact-type property of admitting only constant complex-valued holomorphic mappings. We conclude this section of the thesis by examining the concrete example $C(X)$ for X a compact subset of $\mathbb{R}$ and showing that the dual manifold in this case is given by $C(X, \overline{C})$.

The last chapter of the thesis examines the holomorphic curvature of the tangent norm of an arbitrary dual symmetric manifold (the tangent norm is a Finsler metric and holomorphic curvature is therefore different than in the Riemannian sense). We find that the dual manifolds have constant positive holomorphic curvature.

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Abstract of Doctoral Thesis

A NUMERICAL STUDY OF THE NON-LINEAR BAROTROPIC INSTABILITY OF FREE ROSSBY WAVES AND TOPOGRAPHICALLY FORCED PLANETARY WAVES

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This thesis was prepared in the NIHE, Limerick (University of Limerick) under the direction of Professor P. F. Hodnett and was submitted for the award of Ph.D. to the University of Limerick, July 1990.

The stability of free and forced planetary waves on a β-plane is investigated by integrating numerically the nonlinear quasi-geostrophic, barotropic vorticity equation on a grid-point model. It is shown that the exponential growth rate predicted by the linear model of Lorenz (1972) is accurate. However it is also shown that instability can occur for wave amplitudes $A < A_c$ in the non-linear case where A_c is calculated using the linear model. When stability occurs for large A the perturbation undergoes exponential growth followed by a bounded oscillating behaviour. For small A the perturbation follows an oscillating pattern of growing and subsiding slowly over a long time period. This appears to confirm the analysis of Deininger (1982) and Deininger and Loesch (1982).

The effect of boundary conditions on stability is investigated by comparing the instability for a Rossby wave on a doubly periodic domain with the instability of exactly the same wave in channel. It is found that there is no significant effect on Rossby wave stability. The effect of changing the y dependence on the stability of a Rossby-Haurvitz in a channel is also investigated.

The nonlinear instability of topographic planetary waves on a doubly periodic *beta*-plane as well as in a channel is examined. In the former case an example of a system going form one equilibrium