where

6.2 $$J(w_0 + \sum_{r=1}^{3} \sum_{|j|=r} w_j dx_j) = w_0(0)\underline{1}.$$

Proof. Note the commutation rules

$$D_i D_j - D_j D_i = S_i S_j - S_j S_i = L_i L_j - L_j L_i = 0 \; ;$$

6.3 $$\sum_{i=1}^{3} L_i S_1 D_i = I - J \; ; \; \sum_{i=1}^{3} L_i S_{k+1} D_i = I - k S_k \text{ if } k \geq 1 \; ;$$

$$D_i L_j = \delta_{ij} I + L_j D_i \; ; \; D_i S_k = S_{k+1} D_i \; ; \; L_j S_{k+1} = S_k L_j \; .$$

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ON SOME MATHEMATICAL WORKS IN THE LIBRARY OF THE ROYAL IRISH ACADEMY.

G.L. Huxley, M.R.I.A.

In the course of two centuries the Academy's library has grown steadily. In Irish subjects it has become one of the best collections of manuscripts and printed matter in the country. Other fields of knowledge are also well represented: this short paper draws attention to our mathematical holdings in the hope that mathematicians and historians of mathematics will be encouraged to make greater use of the books and periodicals at Academy House.

Most of the mathematical texts and periodical articles have been obtained by gift or by exchange. Consequently coverage of domains within the subject is far from complete or coherent. Many areas are, however, represented, and the geographical range of periodicals is remarkable. There are long runs of the *Acta* and other serial publications of leading European academies—for example, from Paris, Berlin, Rome, Göttingen, Heidelberg, and Helsinki. Our holdings of the *Philosophical Transactions* of the Royal Society of London extend back to Volume 1 (1665/6). The St. Petersburg *Commentarii* begin in the age of Euler and the set of Liouville's *Journal de Mathématique pures et appliqueés* continues until 1924 from the first issue in 1836. There are strong collections of Japanese periodicals published in English, among them numerous editions of *Tensor* and the *Hiroshima Mathematical Journal*. Current work in the United States is well represented: we receive the *Proceedings of the National Academy of Sciences*, the Princeton *Annals of Mathematics*, the *Duke Mathematical Journal*, and, among other leading periodicals, *Studies in Applied Mathematics* (Cambridge, Massachusetts).

From Eastern Europe come *Colloquium Mathematicum* (Warsaw), *Acta Mathematica Hungarica* (Budapest), the *Czechoslovak Mathematical Journal*; and our intakes from the U.S.S.R. and associated territories are substantial. Outstanding are the *Doklady* of the Academy of Sciences (Moscow and Leningrad) and the Academy's Mathematical *Isvestiya*.

From Georgia and Armenia numerous publications have been received. Irish exchanges with the University of Kazan are of long standing. Matter arrives steadily from Holland and New Zealand, from Spain and from India, and from many other countries. The accessions reflect the international standing of the Academy and the wide distribution of section A of the *Proceedings*. A cherished property is a series of Crelle's *Journal* from Volume 1 (1836) to 144 (1914).

There are few modern mathematical textbooks in the library, but mathematical classics are of permanent value—and not only to historians; and our assemblages benefited greatly from the bequest in 1987 of modern works belonging to the late Professor J. G. Semple. The many books of Geometry reflect the strength of Ireland in the subject in the nineteenth century. The powerful mind of George Salmon is well attested by treatises—for instance, *A Treatise on Higher Plane Curves* (second edition, Dublin 1873); we possess a signed third edition of his *Treatise on the Analytic Geometry of Three Dimensions* (Dublin 1874) and a sixth edition of his *Treatise on Conics* (London 1879). Euclidean Geometry appears in a classic historical exposition by G.J. Allman, *Greek Geometry from Thales to Euclid* (Dublin and London 1889). John Casey's *The First Six Books of the Elements of Euclid* (Dublin and London 1882) contains at p. 249 M'Cullagh's proof of the minimum property of the line named after Philo of Byzantium (expressing two mean proportionals between two given lines). Casey's *Sequel* to the previous work (fourth edition, Dublin and London 1886) exhibits a rigour such as would be welcome in some more recent school-geometries. The Academy possess a copy of G. Monge's *Géometrie descriptive* (fourth edition, Paris 1820) and a Russian translation, with commentary, of the same treatise by A.I.Kargin (Moscow and Leningrad 1947).

Also present is A.M.Legendre's *Éléments de Géometrie* (twel-

fth edition, Paris 1823). The history of non-Euclidean Geometry can be studied in several works: we have V.F.Kagan's biography of Lobachevsky (Moscow and Leningrad 1944); Kagan also edited a selection of Lobachevsky's studies in the theory of parallel lines (*Issledovania*, M.-L. 1945). A *Libellus* (Claudiopolis (Clug) 1902) commemorating the centenary of the birth of the younger Bolyai includes an *index* (by R. Bonoia) *operum ad Geometriam absolutam spectantium*. Among the books of the Semple bequest is H.S.M. Coxeter, *Non-Euclidean Geometry* (Toronto 1942).

The holdings in logic and foundations are significant. Aptly prominent is George Boole: we have a reprint of his *Mathematical Analysis of Logic* (Oxford 1951, originally Cambridge 1847) and a collection, edited by R. Rhees, of Boole's studies in logic and probability (London 1952). Instructive also for historians is Desmond MacHale, *George Boole. His Life and Work* (Dublin 1985). In the same domain is Kurt Gödel, *On Formally Undecidable propositions of Principia Mathematica and Related Systems* (English translation, Edinburgh and London 1962). Another classic of mathematical logic is Boole's *An Investigation of the Laws of Thought*; the Academy is fortunate to possess a first edition (London 1854). Jan Lukasiewicz lectured in the Academy from 1946 onwards on mathematical logic: we have his *Aristotle's Syllogistic from the Standpoint of modern Formal Logic* (Oxford 1951).

Historians of the theory of numbers will find much of interest in the Library. A rare work is B.N. Delone, *The Petersburg School of the Theory of Numbers* (Moscow and Leningrad 1947). Another valued property is the collected *Oeuvres* of P.L.Tchebychef in two volumes (St. Petersburg 1907 and 1899). (The Academy's Russian links, of long standing, are indicated also by the presence of L.Euler's *Opuscula Analytica* I (Petropoli 1783)). Happily the Library has Euler's three volumes of *Dioptrica* also published at St. Petersburg (1769, 1770, 1771). Not only antiquarians will be pleased to find a copy of C.F.Gauss, *Disquisitiones Arithmeticae* (Leipzig 1801) and, perhaps, not only analysts will study with pleasure the papers on Abelian functions and on differential equations in Karl Weierstrass, *Mathematische Werke* I (Berlin 1894); but few nowadays are likely to find easy the notation in Edward Waring's *Miscellanea Analytica de Aequationibus Algebraicis et*

Curvarum Proprietatibus (Cambridge 1762). The translation into Russian of mathematical treatises by Al-Farabi and their publication in Kazakhstan at Alma Ata in 1972 serve to emphasize that the study of the history of mathematics knows no frontier.

A few details in conclusion, chosen not quite at random from selective explorations, will show that the intellectual profit from the use of the Library can be great. Here is C. Huygens studying the ancient problem of the quadrature of the circle (see *Oeuvres Complètes*, Volume 20, The Hague 1940). Here is evidence that Newton supposed the Creator to have made parts of absolute space impenetrable (see his *Opuscula*, Volume I, ed. J. Castillioneus, p.xxxiii, Lausanne and Geneva 1734). Here is Salmon handsomely giving credit to Boole for his part in originating the principles of linear transformation in modern algebra (George Salmon, *Lessons introductory to the Modern Higher Algebra*, third edition, Dublin 1876, p.103). Here is a copy of the third edition of Newton's *Principia* that once graced a library in Ballinlough. And here are collected works of W. Rowan Hamilton, J.J. Larmour, F. Severi and others.

Mathematical practitioners who wish to study in the Library are invited to ask the staff about rules and registration. The Academy believes that the mathematical aspects of its Library deserve to be better known; accordingly an increase in the number of mathematically interested readers would be a welcome development.

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Note. I thank Professors J. T. Lewis and Anthony O' Farrell, both Members of the Academy, for helpful conversation and advice before the writing of this paper.

PROBABILITY IN FINITE SEMIGROUPS

Desmond MacHale

Let S be a finite non-empty set and let $*$ be a closed binary operation on S. For $x \in S$ let $C(x) = C_S(x)$, the centralizer of x in S, be $\{y \in S | x * y = y * x\}$, the set of all elements of S which commute with x. We define $Pr.(S)$ to be $\sum_{x \in S} |C(x)|/|S|^2$ so that $Pr.(S)$ is the probability that a pair of elements of S, chosen at random, will commute with each other.

Clearly, for x, $y \in S$, $x \in C(x)$, and $x \in C(y)$ if and only if $y \in C(x)$, but apart from these trivial restrictions there are no other restrictions on the values $Pr.(S)$ may have. Thus $1 \geq Pr.(S) \geq 1/|S|$ and the size of $Pr.(S)$ is a good indication of "how commutative" $\{S, *\}$ is, since $Pr.(S) = 1$ if and only if S is commutative.

If $\{G, *\}$ is a group then there are severe restriction on the values that $Pr.(G)$ may assume. For example we have the following (see [1] and [2])

(i) If $Pr.(G) > \frac{5}{8}$ then $Pr.(G) = 1$.
(ii) If $Pr.(G) > \frac{1}{2}$ then $Pr.(G) = \frac{1}{2} + \frac{1}{2^{2k+1}}$ for some k.
(iii) It is not possible to have $\frac{7}{16} < Pr.(G) < \frac{1}{2}$.

The bound given in (i) is the best possible and is attained for example by D_4, the group of all symmetries of the square.

At the lower end of the scale it is possible to make $Pr.(G)$ as small as we please in absolute terms, though not as small as $1/|G|$ unless G is trivial.