

The Annual September Meeting of the IMS

University College Cork

27–28 August 2015

Thursday 27 August	
10.30–11.00	IMS opening
11.00–12.00	Stanley Burris
12.00–13.00	Barbara Fantechi
13.00–14.00	<i>Lunch</i>
14.00–15.00	Rajarshi Roy
15.00–15.30	Adele Marshall
15.30–16.00	David Henry
16.00–16.30	<i>Coffee & Tea</i>
16.30–17.00	Rupert Levene
17.00–17.30	Ivan Todorov
17.30–18.30	Chris Rogers
Friday 28 August	
09.00–10.00	Alice Niemeyer
10.00–10.30	<i>Coffee & Tea</i>
10.30–11.00	Aoife Hennessy
11.00–11.30	Boole2School
11.30–13.00	IMS AGM
13.00–14.00	<i>Lunch</i>
14.00–15.00	Franz Pedit
15.00–16.00	Muriel Medard
16.00–16.30	<i>Coffee & Tea</i>
16.30–17.30	Marius Junge

The meeting was funded in part by the European Mathematical Society, Science Foundation Ireland and University College Cork, in conjunction with the IMS. There were 52 registered participants. The timetable of the talks is above, and the abstracts are given below.

Titles and Abstracts

Stanley Burris — A Primer on Boole’s Algebra of Logic

I will describe how Boole managed to squeeze an algebra of logic into the algebra of numbers. The only background needed is high school algebra and the distributive law from basic set theory: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. With this setup Boole proved his elegant results on Elimination and Solution. Boole’s approach led to his models being partial algebras since addition and subtraction were only partially defined, and this led to his use of uninterpretables. Boole’s main results were admired, but not his algebra-of-numbers approach. Within a decade of his publication of Laws of Thought his system was being replaced by the modern Boolean algebra which had total algebras and was able to derive the same results.

Barbara Fantechi — Counting Curves on Algebraic Varieties

Enumerative geometry is one of the oldest parts of mathematics, with entry-level problems that can be explained to the layman (given four lines in space, how many lines meet/intersect all of them?). Yet in recent decades stunning progress has come thanks to a synergy of algebraic and complex geometry, string theory and function theory. In this talk we give a very partial overview, highlighting the interplay between concrete problems and theoretical advances.

Aoife Hennessy — Riordan Arrays and Bijections of Weighted Lattice Paths

This talk introduces Riordan arrays and concerns paths counted by Riordan arrays arising from the decomposition of certain Hankel matrices and bijective relationships between them. We consider certain Hankel matrices, H under two decompositions, $H = L_M D L_M^T$ with L_M a Riordan array of generating functions that count weighted Motzkin paths and a $H = L_L S D S L_L^T$ decomposition, with L_L a Riordan array with generating functions that count weighted Lukasiewicz paths. A bijection is introduced between these paths.

References

- [1] P. Flajolet, Combinatorial aspects of continued fractions, *Discrete Mathematics* **32** (1980), pp. 125–161.
- [2] A. Hennessy, A study of Riordan arrays with applications to continued fractions, orthogonal polynomials and lattice paths, PhD Thesis, Waterford Institute of Technology, 2011.
- [3] F. Lehner, Cumulants, lattice paths and orthogonal polynomials, *Discrete Mathematics* **270** (2003), pp. 177–191.

David Henry — Exact Solutions of the Water Wave Problem

The equations of motion which govern fluid dynamics are highly intractable to mathematical analysis for a variety of reasons, among them being the inherent nonlinearity of the governing system of partial differential equations coupled with the fact that one is often dealing with a free-boundary problem whereby the fluid domain is a priori unknown.

Recently, modern functional analytic techniques have enabled major progress to be made in proving the existence of exact water wave solutions to the fully nonlinear free-boundary value problem. In this talk, which consists of recent work with various collaborators, I will present some exact solutions to the governing equations of water wave problems in two- and three-dimensions which have an explicit form in the Lagrangian formulation, describing briefly how these solutions are particularly amenable to a hydrodynamical stability analysis.

Obligatory Boole reference: one of the major figures in the area of hydrodynamical stability, Sir Geoffrey Ingram Taylor (1886-1975), was none other than George Boole's grandson!

Marius Junge — Analysis on Noncommutative Spaces

Noncommutative tori are simplest examples for noncommutative manifolds in noncommutative geometry. For these concrete spaces and their noncompact analogues, we will discuss basic concepts from noncommutative geometry and finite dimensional approximation in the Gromov–Hausdorff sense. Using a semigroup approach one can show that also many tools in classical harmonic analysis concerning convergence of Fourier series and singular integral operators remain

valid in the context of noncommutative deformations of classical spaces.

Rupert Levene — The Boolean Lattice of Schur Idempotents

A Schur idempotent is an infinite matrix A of ones and zeros with the property that if the matrix of any bounded Hilbert space operator B is changed by replacing b_{ij} with 0 whenever $a_{ij} = 0$, then we obtain another bounded operator. We will introduce these objects and consider an open problem concerning the generation of the Boolean lattice of all Schur idempotents, touching on joint work with Ivan Todorov (QUB) and Georgios Eleftherakis (Patras).

Adele Marshall — Predictive Analytics and Healthcare Modelling Using Coxian Phase-type Distributions

Muriel Medard — Stormy Clouds: Security in Distributed Cloud Systems

As massively distributed storage becomes the norm in cloud networks, they contend with new vulnerabilities imputed by the presence of data in different, possibly untrusted nodes. In this talk, we consider two such types of vulnerabilities. The first one is the risk posed to data stored at nodes that are untrusted. We show that coding alone can be substituted to encryption, with coded portions of data in trusted nodes acting as keys for coded data in untrusted ones. In general, we may interpret keys as representing the size of the list over which an adversary would need to generate guesses in order to recover the plaintext, leading to a natural connection between list decoding and secrecy. Under such a model, we show that algebraic block maximum distance separable (MDS) codes can be constructed so that lists satisfy certain secrecy criteria, which we define to generalise common perfect secrecy and weak secrecy notions. The second type of vulnerability concerns the risk of passwords being guessed over some nodes storing data, as illustrated by recent cloud attacks. In this domain, the use of guesswork as a metric shows that the dominant effect on vulnerability is not necessarily from a single node, but that it varies in time according to the number of guesses issued. We also introduce the notion of inscrutability, as the growth rate of the average number of probes that an attacker has to make, one at a time, using his best strategy, until he can correctly guess one or more secret strings from multiple randomly chosen strings.

Joint work with Ahmad Beirami, Joao Barros, Robert Calderbank, Mark Christiansen, Ken Duffy, Flavio du Pin Calmon, Luisa Lima, Paulo Oliveira, Stefano Tessaro, Mayank Varia, Tiago Vinhoza, Linda Zeger.

Alice Niemeyer — The Divisibility Graph of a Finite Group

For a set of positive integers X , A. Camina and R. Camina introduced the Divisibility Graph of X as the directed graph with vertex set $X \setminus \{1\}$ and an edge from vertex a to vertex b whenever a divides b . For a group G let $cs(G)$ denote the set of conjugacy class lengths of non-central elements in G . Camina and Camina asked how many components the Divisibility Graph of $cs(G)$ has. In joint work with Abdolghafourian and Iranmanesh we determine the connected components of the Divisibility Graph of the finite groups of Lie type in odd characteristic.

The answer to this question is closely related to another kind of graph defined for groups, namely the Prime Graph. The vertex set of the Prime Graph of a finite group G is the set of primes dividing the order of the group and two vertices r and s are adjacent if and only if G contains an element of order rs . Williams investigated the Prime Graph of finite simple groups and determined its connected components.

Franz Pedit — Integrable Surface Geometry for Surfaces of Non-abelian Topology

We give a short historical introduction to the theory of constant mean curvature surfaces and discuss recent advances for surfaces whose fundamental groups are non-abelian. We explain how to use the Riemann-Hilbert correspondence between local systems, representation varieties and holomorphic bundles to give a description of those surfaces in terms of algebro-geometric data. Computer experiments and visualisations support and guide the theoretical investigations making the talk quite accessible to non-experts.

Chris Rogers — Fundamental Fallacies of Finance

The Concise Oxford Dictionary defines a fallacy as “A mistaken belief esp. based on unsound argument”, and the history and current practice of the finance industry provides egregious examples. This talk will fearlessly expose some of these, beat up the unsound arguments, and, unexpectedly, offer some practical suggestions on

how to avoid such errors. These suggestions are unlikely ever to be adopted, for reasons that will also be explained.

Rajarshi Roy — Seeing the Light: Visual Illusions and Reference Frames

The eye and brain work together to provide us with perspectives of reality. Signals from the eye to the brain determine what we see and how we interpret images and the dynamical, changing world around us. We will explore simple and complex aspects of “seeing the light” - from the formation of images to their interpretation based on frames of reference that lead us to impressions of the world around us. Visual illusions and demonstrations with simple apparatus will be used to illustrate how eyes and brain work together to help us navigate our way through life with balance and poise.

Ivan Todorov — Positive Extensions of Schur Multipliers

In this talk I will describe the positive completion problem for matrices, give a summary of known results and then discuss generalisations of these results in infinite dimensions. I will highlight the operator space approach to the topic, in which completely positive maps play a decisive role. Time permitting, I will give some applications of the results to the problem of extending positive definite functions defined on subsets of locally compact groups.

Joint work with R. Levene and Y.-F. Lin,

Report by Stephen Wills, University College Cork