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S. B. G. O'Brien,
Centre for Industrial and Applied Mathematics,
University of Limerick,
Limerick.

THE 35TH INTERNATIONAL MATHEMATICAL OLYMPIAD

Fergus Gaines

The International Mathematical Olympiad (IMO) is the most prestigious mathematical competition in the world for pre-university students. It is held annually and the 1994 contest took place in Hong Kong in July. The number of countries and regions officially participating was 68. Each participating country sent a team of up to six members. The competition consisted of two four and a half-hour examinations, each exam made up of three problems. Each student competed as an individual and medals were awarded to the top performers.

IMO problems are celebrated for their extreme level of difficulty and some of them can even defeat professional mathematicians. It is no surprise, therefore, to find that a young student stands little chance of success in the competition, without a considerable amount of training. Some countries have a whole series of mathematics competitions–one for each year of the school programme–and in this way they can identify and encourage talented students from an early age. The first task in the process of choosing a team to represent Ireland in Hong Kong was to identify suitable candidates for training. Because a certain basis of mathematical knowledge is required in order to benefit from the training programme, generally only students who have completed the Junior Certificate are eligible. In November 1993 most secondary schools were invited to send up to three of their most mathematically talented pupils to attend training sessions in one of UCC, UCD, UCG and the University of Limerick. From information supplied by the Department of Education the top two hundred performers in the 1993 Junior Certificate mathematics examination were also personally invited to attend. The training sessions took place once a week in each of the centres. As well as giving basic instruction in the areas of geometry, inequalities, combinatorics, number theory and algebra, the training sessions also concentrated on problem-solving techniques in these areas. Because the numbers attending in UCD were quite large (about 200, initially) an elimination test was held after four training sessions and the best 30 students were kept on in the training programme there. It was not necessary to have an elimination test in the other centres.

The Seventh Irish Mathematical Olympiad was held on Saturday, 7 May 1994 and consisted of two three-hour examinations. The top six performers in this contest were:

1. John Sullivan, Coláiste an Spioraid Naoimh, Cork.
2. Eoghan Flanagan, St Nessan's Community School, Limerick.
3. Mark Flanagan, St Benildus College, Dublin 14.
4. Deirdre O'Brien, Mount Mercy College, Cork.
5. Richard Murphy, St Munchen's College, Limerick.
6. Mark Dukes, Newpark Comprehensive School, Blackrock, Co Dublin.

and these were invited to form the Irish team for the IMO in Hong Kong. They all accepted the invitation.

A final three-day training camp for the team was held in the University of Limerick from 29 June to 1 July. The training camp is very important for the students–as well as concentrating on problem-solving strategies it gives the students the opportunity to get to know each other well, it helps to generate a team spirit and it helps to increase the motivation of the students to do their best in the competition.

I was the leader of the team and the deputy leader was Donal Hurley of University College Cork. I flew to Hong Kong on Thursday, 7 July and was taken to the Panda Hotel in Kowloon, where the leaders of all the teams were accommodated. The team leaders formed the jury for the final selection of the six problems that were to form the competition. Each participating country had already been invited by the organizers to submit up to six original problems for consideration by the jury. A local committee in Hong Kong had formed a short list of 24 problems from all those submitted. After three days of very long, and sometimes acrimonious, meetings, the six problems for the competition were selected. Although problems submitted are supposed to be original, a number of problems on the list of 24 were rejected because they, or problems very like them, had already appeared in other competitions. There was some dissatisfaction expressed at the quality of the shortlisted problems. Versions of the final six problems were prepared in the four official languages, English, French, Russian and Spanish, and the official text agreed. Finally, translations were made into all the languages required by the students. All the jury meetings took place in the Chinese University of Hong Kong.

The team, accompanied by Donal Hurley, arrived in Hong Kong on Monday, 11 July and were taken to their accommodation in summer camp-style residences in a rural part of Kowloon. There was no contact of any kind between them and the team leader until after the competition. The accommodation was adequate, if a little spartan, and the students found the lack of air-conditioning a bit trying in the humid, summer heat of Hong Kong. They found it difficult to adjust to the Chinese food, but they were able to buy food more to their taste in the local shops and the local McDonalds! The opening ceremony took place on Tuesday, 12 July, performed by the governor of Hong Kong, Mr Christopher Patten. The first exam was held at the Chinese University on Wednesday, 13 July when the first three problems were examined in a four and a half-hour exam. I received the exams of the Irish students that evening and began the work of reading their answers. The second exam was held on the morning of Thursday, 14 July and, that afternoon, all the deputy team leaders moved from their accommodation with the students to join the team leaders in the Panda Hotel.

Donal Hurley and I spent many hours reading the students' work and working out for each student the marks he would be expected to get, based on the marking scheme prepared by the coordinators. We also spent a considerable amount of time pursuing some of the students' lines of thought to see if they would lead to solutions. This needs to be done if a case is to be made for extra marks. To give an idea of the amount of work involved, it happened twice that more than three hours were spent on the work of one student on one problem to get a complete understanding of that piece of work. On Friday and Saturday, 15 and 16 July, we went seven times to the coordinators to agree the marks to be awarded for each question. The seventh trip was needed because John Sullivan had a particularly complicated (and essentially correct) solution to problem no. 6 and a large amount of time was needed to understand his work before it could be presented for coordination. The coordinators for each problem consisted of a group of three local mathematicians. Donal Hurley and I explained, in great detail, what each of the students had done on that problem and agreed, in some cases after much argument, the mark to be awarded to each student.

This year's IMO exam was considered by most observers to be somewhat easier than usual and this was reflected in the high scores of many of the students. The rules of the IMO state that medals can be awarded to at most half of the contestants. It is also stipulated that at most 1/6th of those eligible can receive gold medals and that the corresponding fractions for silver and bronze are 1/3 and 1/2, respectively. A perfect answer to a question gains 7 points and, thus, the maximum number of points that a student can score is 42. Partial credit for a question is awarded, but a student has to do some significant work before any marks at all are given. The marks gained by the Irish students were:

Mark Dukes	11
Eoghan Flanagan	16
Mark Flanagan	10
Richard Murphy	15
Deirdre O'Brien	2
John Sullivan	14

Thus the team score was 68, which meant that Ireland got 49th place out of 69 competing countries. In order to win a bronze medal a contestant had to score at least 19 points, so the Irish won no medals this year. However, since any student who does

not get a medal and who scores 7 points on at least one question gets an "honourable mention", three of the students, Eoghan Flanagan, Mark Flanagan and Richard Murphy received this honour. Question 6 was by far the most difficult on the exam and this fact was underlined by the large number of students who scored zero on it.

There were some surprises in the results of some other countries. In recent years China has become the strongest team in the IMO, so it was quite a surprise when they were beaten into second place by the United States. The U.S. team created a record because every one of their students scored full points and this has never happened previously in the IMO. The Chinese team caused quite a stir when three of their students scored zero on question 6! This makes John Sullivan's mark of 4 on that question all the more meritorious.

The team scores, out of a maximum of 252, for the leading ten countries were:

United States	252
China	229
Russia	224
Bulgaria	223
Hungary	221
Vietnam	207
United Kingdom	206
Iran	203
Romania	198
Japan	180

It would not be possible for Ireland to participate in the IMO without considerable support from many people and organizations. The organizers are extremely grateful to the sponsors for financial and other assistance. The sponsors of the Irish participation in the 1994 IMO were

An Roinn Oideachais
Forbairt
University of Limerick
Arts Faculty, University College Dublin

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I give here the six problems of the 35th IMO. Solutions are given below on pages 74 to 76.

1. Let m and n be positive integers. Let $a_1, a_2 \dots, a_m$ be distinct elements of $\{1, 2, \dots, n\}$ such that whenever $a_i + a_j \leq n$ for some i and j, $1 \leq i \leq j \leq m$, there exists k, where $1 \leq k \leq m$, with $a_i + a_j = a_k$. Prove that

$$\frac{a_1 + a_2 + \cdots + a_m}{m} \geq \frac{n+1}{2}.$$

2. ABC is an isosceles triangle with $AB = AC$. Suppose that
(*i*) M is the midpoint of BC and O is the point on the line AM such that OB is perpendicular to AB;
(*ii*) Q is an arbitrary point on the segment BC different from B and C;
(*iii*) E lies on the line AB and F lies on the line AC such that E, Q and F are distinct and collinear.
Prove that OQ is perpendicular to EF if and only if $QE = QF$.

3. For any positive integer k, let $f(k)$ be the number of elements in the set $\{k+1, k+2, \dots, 2k\}$ whose base 2 representation has exactly three 1's.
(*a*) Prove that, for each positive integer m, there exists at least one positive integer k such that $f(k) = m$.
(*b*) Determine all positive integers m for which there exists exactly one k with $f(k) = m$.

4. Determine all ordered pairs (m, n) of positive integers such that

$$\frac{n^3+1}{mn-1}$$

is an integer.

5. Let S be the set of real numbers strictly greater than -1. Find all functions $f : S \to S$ satisfying the two conditions:
(*i*) $f(x + f(y) + xf(y)) = y + f(x) + yf(x)$ for all x and y in S;

(*ii*) $\frac{f(x)}{x}$ is strictly increasing on each of the intervals $-1 < x < 0$ and $x > 0$.

6. Show that there exists a set A of positive integers with the following property: for any infinite set S of primes there exist positive integers $m \in A$ and $n \notin A$ each of which is a product of k distinct elements of S for some $k \geq 2$.

Fergus Gaines,
Department of Mathematics,
University College,
Belfield,
Dublin 4.