Book Review

A FIRST COURSE IN NONCOMMUTATIVE RINGS

(Graduate Texts in Mathematics)

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Reviewed by Mark Leeney

Lam's book is a welcome addition to the literature on noncommutative rings. The first book of an intended two-volume introduction to ring theory, it is a fleshed-out version of the material covered in a one-semester graduate course designed and delivered to second year graduate students at Berkeley. In the Preface, Lam convincingly pleads the case for ring theory as 'an indispensable part of the education for any fledgling algebraist', citing the connections between it and group theory, functional analysis, algebraic geometry, to name but a few. Given the ubiquitous nature of noncommutative rings it is unfortunate that the number of books written at this introductory level is so few.

As per the origins of the subject, the book opens with the Wedderburn-Artin theory of semi-simple rings and in short order presents an impressive collection of examples. These include the quaternions, free rings, rings with generators and relations, group rings, Laurent series rings, twisted polynomial rings, differential polynomial rings, the tensor algebra of a finite dimensional vector space and triangular rings. These examples form the bedrock for motivation and serve to provide the author with a presumed knowledge on the part of the student which otherwise might not be the case. This is a lot of information to be assimilated by a student and I must admit to questioning the

efficacy of introducing the newcomer to so many new 'faces' at once. Chapter 2 continues with the Jacobson radical and group rings and the J-semisimplicity problem. Modules over finite dimensional algebras, representations of groups and linear groups are considered in Chapter 3. Chapter 4 introduces the prime radical and prime/semiprime rings. The structure of primitive rings, the Density theorem, subdirect products and the commutativity theorems are also presented there. Chapter 5 embarks on division rings; along the way proving Wedderburn's, Frobenius's and Cartan-Brauer-Hua's theorems. Explicit constructions of division rings further increases the stock of examples. Tensor products, maximal subfields and polynomials over division rings are also treated. Chapter 6 follows the generalization of the Artin-Schrier criterion for the existence of orderings on fields to rings without zero-divisors. As a special case, ordering structures on division rings are considered. Chapter 7 focuses on local/semilocal rings and idempotents. Standard results on lifting idempotents modulo an ideal are developed and used to study the Krull-Schmidt decomposition of modules. The final chapter deals with perfect and semiperfect rings and homological characterizations thereof. The notion of the basic ring of a semi-perfect ring is considered and is brought to bear on right artinian rings, culminating with examples of principal indecomposable modules and Cartan matrices.

As is customary with the series, the text is well bound, uncluttered in appearance and conventional in notation; typos are conspicuous only by their absence. The style is fairly informal without being chatty. Each chapter begins with a useful introduction to the material about to be presented; how the material developed historically, its power and limitations, and the goals of the chapter.

Sets of exercises appear at the end of each section of each chapter, thereby reinforcing the material on a regular basis. Exercises are, generally speaking, straightforward tests of understanding of the concepts introduced in the text. Trickier ones are supplied with hints and serve as a useful complement to the theory and examples in the text. An especially endearing aspect of the book is Lam's occasional excursions into 'open problem' territory. At such locations, time is spent in discussing the current state of affairs relating to a problem, partial answers and possible approaches : a stimulating insight into the process of doing research-level mathematics is given.

The book, generally, has a good balance of theory and examples. However, the initial stockpile of twenty-six examples in the first twenty-three pages may prove a bit intimidating. For reference purposes (and revisits are quite likely) the convenience of having these examples en bloc will be welcome - if one gets past page twenty-three with the desire to continue still intact. The no-nonsense, go-ahead style of the book is in keeping with the rest of the series and its setting of material in historical context together with the tantalizing references and discussion of open questions should have the desired effect of exciting and encouraging the student. However, I do wonder how plausible it would be to try to cover all this material in one semester. Of the many sins a teacher can commit, two spring to mind. Of these two, the worse is to underestimate the ability of one's students - this smacks of condescension and pride. The lesser evil is to overstretch a class - this can be construed as overindulgence, nay even one-upmanship. If Lam is guilty of such a transgression, at least he has chosen the lesser evil.

Any serious mathematical library should have this book in its collection. It could be used to good purpose as the basis for a one-year excursion into the realms of noncommutative rings or, as a text for self study by mathematically mature students. Like a good meal, this first course should serve as a satisfying entrée, leaving one wanting more. It does.

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