PROBLEM PAGE

Editor: Phil Rippon

Just two new problems this time. The first is an inequality which is elementary, in the sense that it can be solved easily by rearrangement to give a perfect square. However, there is a much more interesting solution using an approach which makes the inequality 'obvious'!

23.1 *Find a context within which the inequality*

$$\frac{(a+b)(c+d)}{a+b+c+d} \geq \frac{ac}{a+c} + \frac{bd}{b+d}, \quad a,b,c,d > 0,$$

is intuitively obvious.

I heard the next problem from several tutors at an O.U. Summer School; apparently, it has became a popular 'investigation' at teacher training colleges. Roughly speaking, the problem is to find the number of triangles which have integer sides and perimeter n.

23.2 *Let $s(n)$ denote the number of triples (a,b,c), where a,b,c are positive integers with*

$$a \leq b \leq c \text{ and } a+b > c.$$

Determine a simple formula for $s(n)$.

Now here are the solutions to two earlier problems, which I described as geometric doodling.

21.1 *What is the minimum number of (strictly) acute angled triangles into which a square can be partitioned?*

The solution is given in Fig. 1, which contains three construction lines (one straight line and two semi-circles).

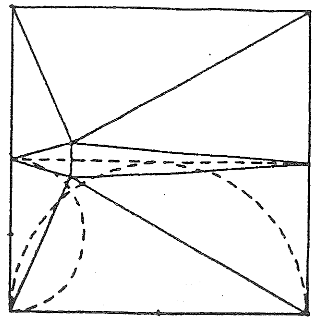

Fig. 1

This problem appears in H.S.M. Coxeter's book 'Introduction to Geometry' and I think it was discussed by Martin Gardiner in Scientific American in the sixties. It must surely be impossible to find a solution with fewer than 8 triangles, but I don't know a proof. It is possible to solve the problem with any $n \geq 8$, however, as Fig. 2 shows (it is of course easy to go from n to $n+3$).

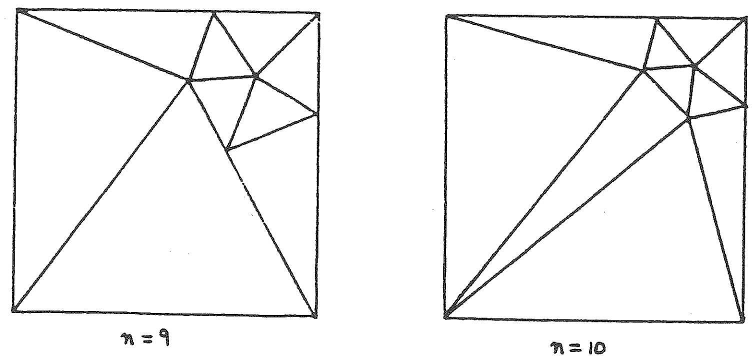

Fig. 2

21.2 *Find a configuration of finitely many points in the plane such that the perpendicular bisector of the line segment joining each pair of points passes through at least two of the points.*

The solution is to take the points to be the vertices of four equilateral triangles built on the sides of a square. It takes only a little work to check that this configuration has the required properties.

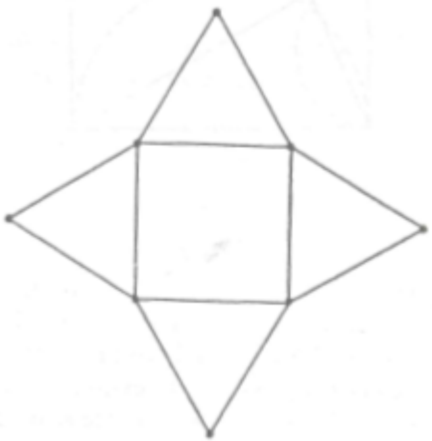

Fig. 3

Thanks to H.S.M. Coxeter and W. Moser, I have been in touch with L.M. Kelly of Michigan State University who considered this problem back in 1964 when he was visiting the University of Cambridge. He believes that the problem may have originated with Paul Erdős and thinks that it is still unknown whether this is the unique such configuration. The nearest I have come to finding another solution was to replace the four equilateral triangles in Fig. 3 by their reflections in the corresponding sides of the square. I'll let you discover why my momentary excitement was quickly followed by disappointment!

Phil Rippon
Faculty of Mathematics
The Open University
Milton Keynes MK7 6AA, UK

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