AN INTRODUCTION TO ALGEBRAIC TOPOLOGY

by Joseph J.Rotman, Graduate Texts in Mathematics 119, Springer-Verlag, 1988, 433pp., ISBN 0-387-96678-1

This is a well written, often chatty, introduction to algebraic topology which "goes beyond the definition of the Klein bottle, and yet is not a personal communication to J.H.C. Whitehead." Having read this book, a student would be well able to use J.F.Adam's "Algebraic Topology:A Student's Guide" to find direction for further study. The book begins with a sketch proof of the Brouwer fixed point theorem: if $f : D^n \to D^n$ is continuous, then there is an $x \in D^n$ such that $f(x) = x$. Functorial properties of homology groups imply that the sphere S^n is not a retract of the disc D^{n+1}, and then a simple argument by contradiction shows that f must have a fixed point. This illustrates the basic idea of studying topological spaces by assigning algebraic entities to them in a functorial way. There follows a rigorous account of the singular homology of a space which assumes only a modest knowledge of point-set topology and a familiarity with groups and rings. The account includes the Hurewicz map from the fundamental group to the first homology group, and ends with a proof of the Mayer-Vietoris sequence. By page 110 a complete proof of Brower's theorem has been given.

Singular homology is good for obtaining theoretical results, but not so good for computations. So simplicial homology is introduced in Chapter 7, and used to compute the homology groups of some simple spaces such as the torus and the real projective plane. A proof of the Seifert-Van Kampen theorem for polyhedra is given at the end of the chapter. Continuing the search for effective means of computing homology groups, Chapter 8 introduces CW complexes and their cellular homology. Chapter 9 begins with a statement (without proof) of the axiomatic characterisation of homology theories due to Eilenberg and Steenrod, and then introduces enough homological algebra to prove the Eilenberg-Zilber theorem and Künneth formula for the homology of a product of spaces. Chapter 10 deals with covering spaces. The higher homotopy groups are studied in Chapter 11 using the suspension and loop functors. Results obtained include the exact homotopy sequence of a fibration, and its application to the fibration $S^3 \to S^2$ to show that the group $\pi_3(S^2)$ is non-trivial. The isomorphism $\pi_3(S^2) \equiv Z$ is beyond the scope of the book. In the final chapter a short discussion on de Rahm cohomology is used to motivate the study of the cohomology ring of a space.

The book is nicely structured, with explanations of where the theory is heading given at frequent intervals. Important definitions are often accompanied by a discussion on their origins. Many exercises are given at the end of sections. Proofs are usually given in full detail. Even though probably every result in the book (and many more besides) can be found in E.H. Spanier's classic text "Algebraic Topology", J.J.Rotman's style of exposition makes the book a useful reference. However a lecture course based on this book may turn out to be a bit slow and dry. (Unfortunately the book corresponds to the syllabus of a one year course given at the University of Illinois, Urbana.) For example the homology of a space is defined on page 66 but we have to wait until page 157 until the homology of the torus is calculated, and until page 226 for the homology of a lens space. The fundamental group is introduced on page 44 but is not calculated for a wedge of two circles until page 171. Maybe too much rigour and generality in a first course on any topic is not a good thing!

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