

The following table records, for $1 \leq n \leq 10$, the calculated values of $T(2)$ and $T(3)$, of $\Omega(3)/2!$ and $\Omega(7)/3!$, and of the probabilities p_2 and p_3 that a randomly chosen pair or trio of sets is non-degenerate, recorded to four decimal places. It shows as expected that the probabilities, though small for small values of n , rise as n does. Better information on their behaviour for large n is easy to extract from the above formulae, which yield that

$$p_2 = 1 - 3(3/4)^n(1 + o(1))$$

$$p_3 = 1 - 7(7/8)^n(1 + o(1))$$

and in general, where p_s is defined as $\Omega(2^s - 1)/s!T(s)$, that

$$p_s = 1 - (2^s - 1)(1 - 2^{-s})^n(1 + o(1))$$

as $n \rightarrow \infty$. Since these probabilities tend to 1, we are obliged to concede that they told us the truth all those years ago. Rather a pity, really.

| n | $T(2)$ | $\Omega(3)/2!$ | p_2 | $T(3)$ | $\Omega(7)/3!$ | p_3 |
|-----|---------|----------------|-------|-------------|----------------|-------|
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 6 | 0 | 0 | 4 | 0 | 0 |
| 3 | 28 | 3 | .1071 | 56 | 0 | 0 |
| 4 | 120 | 30 | .2500 | 560 | 0 | 0 |
| 5 | 496 | 195 | .3931 | 4,960 | 0 | 0 |
| 6 | 2,016 | 1,050 | .5208 | 41,664 | 0 | 0 |
| 7 | 8,128 | 5,103 | .6278 | 341,376 | 840 | .0025 |
| 8 | 32,640 | 23,310 | .7142 | 2,763,520 | 30,240 | .0109 |
| 9 | 130,816 | 102,315 | .7821 | 22,238,720 | 630,000 | .0283 |
| 10 | 523,776 | 437,250 | .8348 | 178,433,024 | 9,979,200 | .0559 |

Department of Pure Mathematics
The Queen's University of Belfast
Belfast BT7 1NN, Northern Ireland

HISTORY OF MATHEMATICS

Giovanni Frattini 1852–1925

Maurizio Emaldi

(Communicated by M.L. Newell)

A little over one hundred years ago, between 1885 and 1886 three papers by the Roman mathematician, Giovanni Frattini "On the generators of groups of operations" appeared in the proceedings of the Royal Academy of Lincei. In the first of these the author introduced the subgroup Φ of a finite group of operations consisting of the set of all operations which "cannot effectively contribute to the generators" of the group. This can be characterized as the intersection of all proper maximal subgroups. He demonstrated that the group in question is nilpotent and in doing so used a most elegant argument which today is called "the Frattini argument". The results contained in these three papers, the full scope of which were not fully grasped at the time of their publication, are amongst the most significant contributions of Italian mathematicians to the theory of groups in the latter half of last century. The definition of the subgroup Φ of a finite group given by Frattini has been extended to groups in general and today is generally called "the Frattini-subgroup". (As far as we can determine, this name appeared explicitly for the first time in a paper by G. Zacher: "Construction of finite groups with trivial Frattini-subgroup." Rend. Sem. Mat. Padova, vol. 21, 1952). In group theory the Frattini-subgroup and more generally the analogous notion in algebraic structures play a central role in many questions. Thus it seems opportune to give a brief biography of the author and document his mathematical interests. While our investigations have led to a complete list of his publications, we shall give but a selection here. We have used the writings of R. Marcolongo "Bollettino di Matematica (1926)", of P. Teofilato "Memorie della Pontificia Accademia dei Nuovi Lincei (1926)", G. Zappa "Supplemento ai Rendiconti

del Circolo Matematico di Palermo (1985)" and research on Frattini undertaken by O. Vanuzzo in 1982/3. Furthermore, we have availed of the help of other people, notably A.C. Garibaldi of Genoa.

Biographical Notes

Giovanni Frattini was born in Rome on the 8th of January, 1852 and in that city he did all of his studies. He was admitted in November 1869 to the first year course in Mathematics and in June 1870, passed his Bachelor's examination with honours. His tutors at University were Battaglini, Beltrami and Cremona and in July 1875, he was awarded his degree in Mathematics. In 1876, he was in charge of Mathematics in the Liceo di Caltanissetta and from there, in November 1878, was transferred to the Technical Institute of Viterbo. In 1879 he became principal teacher of Mathematics and Descriptive Geometry. In February 1881 he obtained a transfer to the Technical Institute of Rome. He remained in Rome until the end of April 1916, having opted for the position of lecturer in the Military College where he taught right from its foundation in 1884. On the 1st of August 1921, he retired.

He was one of the most senior members of the "Mathesis", an association founded in Rome in 1895 by secondary school teachers of Mathematics aimed at improving the schools and raising teaching standards. Between 1900 and 1902, he held the office of President and although his term was extended for a further two years, he did not wish to accept in order to secure the re-election of the association's first President R. Bettazzi. In 1914, he was awarded lectureship qualification in Algebra at the University of Rome, but never took up an appointment. He was a member of the Mathematical Society of Palermo. In 1917, he became an associate member of the Pontificia Romana Accademia dei Nouvi Lincei and an ordinary member in 1918.

He held office as city counsellor and in this guise, helped in the erection of a monument to his favourite poet G.G. Belli, whose sonnets he was wont to recite to his pupils in perfect Roman dialect. This practice of his led to a ministerial enquiry which ended happily. He declared himself willing to replace these sonnets by the poetry of a highest undersecretary of Education. Without further ado, he was allowed continue his recitation of Belli's poetry. A gifted teacher, he wrote several books for the elementary schools in Rome, for the two year course at the Technical Institute and for the Military College. These concise books, bereft of superfluous abstraction and rich in elegant bril-

liance makes one aware of Frattini's art of teaching. The spirit of the master is evident from the following verse which heads a collection of exercises in Frattini's book "Elementi di Calcolo Letterale" (Paravia, 1908).

— Gino mio, l'ingegno umano
partorì cose stupende,
quando l'uomo ebbe tra mano
meno libri e piu faccende — (Giusti)

implying that greater things are achieved by doing more and reading less.

As a researcher and scientist, Frattini cultivated all the mathematical disciplines and studied with particular benefit differential geometry, the theory of groups and the analysis of second degree indeterminates. On this latter topic, his noteworthy contributions to the *Periodico di Matematica* 1891/92 simplify significantly the classical methods of Euler, Lagrange and Gauss. Two large tracts in 1883 and 1884 on groups of transitive substitutions which were the fruit of his study of the classical work of Jordan and some brilliant works of Capelli, made him eligible for appointment to the Chair of Complementary Algebra in Naples in 1886. Amongst the contestants in this truly famous competition were Capelli, Cesàro and Besso.

Subsequently, however, for family reasons, and because he did not wish to leave Rome, he did not compete for other University posts and remained in secondary teaching, making a magnificent contribution "turning the advances of science to the benefit of the school". This motto of Frattini's was imprinted on the journal "*Il Bollettino di Matematica*" founded by A. Conti at Bologna in 1902.

The last years of his life were not happy ones. He was deeply troubled by the war and further by the illness of one of his sons, a war invalid, the loss of his wife and the difficult economic conditions which forced him to work even though he had left teaching for some years. All of this left him dejected but he still managed to retain his old habit of frequenting his favourite Roman cafe where as he often pronounced he "held his chair". He died on the 21st July, 1925, while acting as supervisor of examinations for the School Leaving Certificate of the Liceo Scientifico di Roma.

Apart from 21 papers in the "*Periodico di Matematica*", 6 articles in "*Il Bollettino di Matematica*", 3 contributions to "*La Matematica Elementare*" and over 13 general publications on mathematical topics, some written in witty poetic form - his main works are the following:

Memorie R. Accademia dei Lincei

- 1883 I gruppi transitivi di sostituzioni dell'istesso ordine e grado. Serie III, Vol. XIV, pagg. 143-172.
- 1884 Intorno ad alcune proposizioni della teoria delle sostituzioni. Serie III, Vol. XVIII, pagg. 487-513.

Rendiconti R. Accademia dei Lincei

- 1883 I. Gruppi a K dimensioni. Serie III, Vol. VIII, pagg. 260-264.
- 1885 Intorno a un teorema di Langrange. Serie IV, Vol. pagg. 136-142.
- 1885 Un teorema relativo al gruppo della trasformazione modulare di grado p . Nota I, Nota II, Serie IV, Vol. I, pagg. 142-147, 166-168.
- 1885 Intorno alla generazione dei gruppi di operazioni. Serie IV, Vol. I, pagg. 281-285.
- 1885 Intorno alla generazione dei gruppi di operazioni. Nota II. Serie IV, Vol. I, pagg. 455-456.
- 1886 Intorno alla generazione dei gruppi d'operazioni e ad un teorema d'Aritmetica. Serie IV, Vol. II, pagg. 16-19.
- 1886 Estensione ed inversione d'un teorema d'Aritmetica. Serie IV, Vol. II, pagg. 132-135.
- 1892 Due proposizioni della teoria dei numeri e loro interpretazione geometrica. Serie IV, Vol. I, pagg. 51-57.
- 1892 A complemento di alcuni teoremi del sig. TCHEBICHEFF. Serie V, Vol. I, pagg. 85-91.
- 1893 Di un doppio isomorfismo nella teoria generale delle sostituzioni. Serie V, Vol. II, pagg. 253-259.
- 1903 Di un gruppo continuo di trasformazioni decomponibili finitamente. Serie V, Vol. XII, pagg. 74-82.

Atti Della Pontificia Accademia Romana dei Nuovi Lincei

- 1917 Di una dualità reciproca tra coppie di quadrilateri inscritti nel medesimo cerchio. Sessione VII, pagg. 136-139.
- 1918 Intorno a una questione di minimo relativa alle equazioni. Sessione VI, pagg. 210-219.
- 1920 Alcune considerazioni poligono vettoriali. Sessione I, pag. 25.
- 1922 Una formula di approssimazione relativa al gruppo della radice quadrata. Sessione IV, pagg. 99-104.
- 1923 Un problema di ampliamento per isomorfismo cui dà luogo la teoria della relatività. Sessione III, pagg. 94-99, Sessione V, pagg. 146-156.
- 1924 La relatività e le frazioni continue. Sessione II, pagg. 80-81.
- 1924 Intorno alla proprietà caratteristica dei numeri primi. Sessione III, pagg. 103-105.
- 1924 Intorno a una proprietà caratteristica delle funzioni intere d'una variabile. Sessione VII, pagg. 174-178.

*Dipartimento di Matematica Pura e Applicata
Università di Padova*