A first chapter is provided which contains sufficient concepts from abstract fuzzy topology to make the book self-contained.

"ABELIAN VARIETIES" (Second Edition)

By *D. Mumford*

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This book is a systematic account of the basic results about abelian varieties. It includes an exposition, on the one hand, of the analytic methods and results applicable when the ground field k is the complex field C and, on the other hand, of the scheme-theoretic methods and results used to deal with inseparable isogenies when the ground field k has characteristic p.

The revised second edition contains, in addition, appendices on "The Theorem of Tate" and the "Mordell-Weil Theorem".

PROBLEM PAGE

First item this time is one of those intriguing problems which is often solved more easily by non-mathematicians! I heard it from Richard Bumby who traces it back to John Conway.

1. Find the next entry in the following sequence:

$$1,\ 11,\ 21,\ 1211,\ 111221,\ 312211,\ \ldots\ .$$

Here is another problem with a simple solution which is not so simple to discover.

2. Find an infinite family of pairs of distinct integers m,n such that:

m,n have the same prime factors, and

m-1, n-1 have the same prime factors.

Now for the solutions to some earlier problems, from March 1986.

1. How long is the recurring block of digits in $(0.\dot{0}0\dot{1})^2$?

I first heard this problem from David Fowler, who uses it as an example to show that simple arithmetic can be surprisingly tricky.

Many people's first guess at the answer is 6 digits or 9 digits, but in fact the recurring block has 2997 digits! To be precise:

$$(0.\dot{0}0\dot{1})^2 \quad = \quad 0.\dot{0}00001002\ \ldots\ 99699799\dot{9}.$$

In case you think that there is a misprint here, the string 998 is indeed absent.

Once the pattern in this recurring block has been noticed it is not hard to show that

$$(0.\underbrace{\dot{0}0 \ldots 0\dot{1}}_{n})^2 = \frac{1}{(10^n-1)^2}$$

has a recurring block of $n(10^n-1)$ digits. To verify that the decimal expansion has the form

$$(0.\underbrace{\dot{0}0 \ldots 0\dot{1}}_{n})^2 = 0.\underbrace{\dot{0}0 \ldots 0}_{n}\underbrace{00 \ldots 1}_{n} \ldots \underbrace{99 \ldots \dot{9}}_{n}$$

with the string 99 ... 8 missing, one can apply the identity

$$\frac{10^n(m(10^n-1)+1)}{(10^n-1)^2} = m + \frac{(m+1)(10^n-1)+1}{(10^n-1)^2}$$

with $m = 0,1, \ldots, 10^n-2$, in the long division $1/(10^n-1)^2$.

2. Prove that at least one of the numbers

$$\pi + e, \quad \pi e$$

is transcendental.

Thanks to Des MacHale for supplying this problem and its solution.

We use the facts that if x and y are both algebraic numbers then so are $x \pm y$ and xy (see Herstein's Topics in Algebra, page 172), and also $\sqrt{|x|}$. Thus if both $\pi + e$ and πe are algebraic we deduce that

$$\pi = \tfrac{1}{2}(((\pi+e)^2 - 4\pi e)^{\frac{1}{2}} + (\pi+e))$$

is algebraic, which is clearly false.

The argument clearly holds for any pair of transcendental numbers α, β and Des points out that there are generalisations

to more than two numbers, involving the symmetric functions.

3. Suppose that $a_n \geq 0$, for $n = 1,2, \ldots$. How large can

$$\sum_{n=1}^{\infty} \frac{a_n}{e^{a_1+a_2+\ldots a_n}} \qquad (*)$$

be?

Tom Carroll (a postgraduate at the OU) recently encountered a series of this form while constructing a certain subharmonic function.

In fact the series is convergent with sum less than 1. One can see this by noting that

$$\frac{a_n}{e^{a_1+a_2+\ldots+a_n}} \leq \frac{e^{a_n}-1}{e^{a_1+a_2+\ldots+a_n}}$$

$$= \frac{1}{e^{a_1+a_2+\ldots+a_{n-1}}} - \frac{1}{e^{a_1+a_2+\ldots+a_n}},$$

since $e^x \geq 1+x$. Thus, by telescoping cancellation, the nth partial sum of (*) is at most

$$1 - \frac{1}{e^{a_1+a_2+\ldots+a_n}} < 1.$$

To see that the number 1 is best possible here, consider

$$\sum_{n=1}^{\infty} \frac{a}{e^{na}} = \frac{a}{e^a - 1}, \quad a > 0,$$

and notice that

$$\lim_{a \to 0} \frac{a}{e^a - 1} = 1.$$

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